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A NEW METHOD FOR SOLVING SURFACE-PIERCING-STRUT PROBLEMS.(U)

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A NEW METHOD FOR SOLVING SURFACE-PIERCING-STRUT PROBLEMS

Toshimitsu Kaiho

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The problem of a potential flow around a surface-piercing strut at a high Froude number is proposed. The strut can be of asymmetric cross-section and is not necessarily slender. The exact boundary-value problem is formulated and is approximated by a mathematical model which satisfies the linearized free-surface boundary conditions on the undisturbed free-surface and the rigid-body boundary condition on the actual surface of the strut.

It is found that the problem can be broken into two parts, one of which is the conventional infinite-fluid problem, the other a problem which can be

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formulated as a two-dimensional unsteady-flow problem.

A numerical scheme is developed and applied for three cases, namely, aspect ratios of 0.5, 1.0 and 3.0 . The cross-section of the strut is a lens shape, the thickness/length ratio of which is 0.1. Side-force coefficients and wave patterns are calculated for various Froude number.

The side-force coefficient calculated for 0.5 aspect ratio shows good agreement with the experimental result for a flat plate of the same aspect ratio and angle of attack. The side-force coefficient for aspect ratios 1.0 and 3.0 is shown to change with respect to Froude number in a pattern similar to that of 0.5. Also, it is shown to approach the analytical limit values as the Froude number increases.

The results calculated predict a significant change of the wave pattern on the suction side of the strut as the aspect ratio changes.

The present method can be applied for prediction of the bow wave. The numerical calculation is performed for a double-wedge-shaped model used by Standing (1974). The calculated bow wave pattern shows good agreement with the corresponding experimental result.

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ABSTRACT

The problem of a potential flow around a surface-piercing strut at a high Froude number is proposed. The strut can be of asymmetric cross-section and is not necessarily slender. The exact boundary-value problem is formulated and is approximated by a mathematical model which satisfies the linearized free-surface boundary conditions on the undisturbed free-surface and the rigid-body boundary condition on the actual surface of the strut.

It is found that the problem can be broken into two parts, one of which is a conventional infinite-fluid problem, the other a problem which can be formulated as a two-dimensional unsteady-flow problem.

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AR	aspect ratio
C_ℓ	span-wise side-force-density coefficient
C_L	side-force coefficient
D	depth of the strut
F_n	Froude number
g	gravitational constant
G(P;Q)	Green's function
H	surface of the strut: H = SS + SP + SB
H(X,Y,Z)	function defining the strut surface
L	chord length of the strut
LE	leading edge of the strut
$\underline{n}$	strut surface unit normal vector
n_X, n_Y, n_Z	X,Y and Z component of n
$\underline{N}$	two-dimensional unit normal vector in the cross section of the strut
N_Y, N_Z	Y and Z component of N
SS, SP, SB	starboard-side surface, port-side surface and bottom surface of the strut
S_w	wake
S_w^+, S_w^-	starboard-side surface and port-side surface of the wake
t	nondimensional time
TE	trailing edge of the strut
t_n	n-th time station
TR	thickness/chord length ratio of the strut
U	ship speed (or stream speed)
W(X,Y,Z)	function defining the wake
X,Y,Z	right-handed cartesian coordinates

x,y,z	nondimensional coordinates
α	angle of attack
$\Gamma_D(Z)$	circulation around the strut
$\Gamma(z)$	nondimensional circulation
Δt_n	nondimensional time interval between t_n and t_{n+1}
$\Delta\Phi$	potential jump across the wake
$\Delta\phi$	nondimensional potential jump across the wake
ζ_D	free-surface elevation
ζ	nondimensional free-surface elevation: $\zeta=\zeta_D/L$
κ	wave number: $\kappa=g/U^2$
ξ,η,ρ	nondimensional coordinates
Φ	velocity potential of the perturbation motion
ϕ	nondimensional perturbation velocity potential

I. INTRODUCTION

The potential flow around a surface-piercing strut travelling at a high Froude number is the subject of investigation in this work. The cross section of the strut can be of either symmetric or asymmetric form. The solutions are limited to thin struts with a small degree of lateral asymmetry. Both the leading edge and trailing edge should be sharp. These restrictions are necessary so that the waves generated by the presence of the strut are small and so the free-surface boundary condition may be linearized.

It was probably in a case of surface-piercing hydrofoils (Nishiyama, 1964) that the problem involving lateral asymmetry was studied seriously for the first time. However attempts in this direction did not go beyond lifting-line theory. Newman comprehensively discussed the problem of asymmetrical surface-piercing struts after the fashion of lifting-surface theory. Kern (1973) also used lifting-surface theory and solved the inverse problem. Daoud (1973) succeeded to solve the proposed problem for the case of a yawed surface-piercing flat plate. However Daoud's work shows how complicated it is to solve the two-dimensional singular integral equation of linear lifting-surface theory with the presence of the free surface, for even the simplest case.

In the present work, the boundary-value problem is solved following a method which is related to slender-ship theory.

Tuck (1963) introduced the method of matched asymptotic expansion into ship hydrodynamics to develop his slender-ship theory. Tuck's method made it possible to take full advantage of the slender-ship characteristics. That is, the two-dimensional approximation and the rigid-wall free-surface condition can be applied near the ship provided the Froude number is order of unity. However, the wave resistance according to Tuck's theory becomes infinite except for a ship with a cusped wedge bow or a pointed bow.

Ogilvie (1972) assumed the longitudinal perturbation near the bow of a slender ship (bow near field) to be still small compared with the transverse perturbation but much larger than it is near the center part of the ship. With this assumption he obtained a bow near field where the linearized free-surface condition applied instead of the rigid-wall condition.

The Laplace equation remained two-dimensional. He also used an assumption that the solution is independent of downstream conditions. With these assumptions, he solved the problem in the bow near field. Later, Ogilvie (1975) succeeded in obtaining the far-field solution which matched with the bow-near-field solution near the ends and matched with the usual near-field solution near the middle part of the body. When he applied this modification, the wave resistance from slender-ship theory exactly coincided with the Michel-integral value. His assumption that the longitudinal perturbation is much larger in the bow near field

than it is near the center part of the ship can be understood in that the Froude number in the bow near field is much higher than the usual Froude number, which depends on ship length. This is because the Froude number can be taken as a measure of the relative magnitude of inertial forces with respect to gravitational forces in the interior of a fluid region. His other assumption, stating that the solution is independent of downstream conditions, can be understood more clearly by changing the longitudinal axis into time domain: $X = UT$. Then the problem is reduced to a two-dimensional unsteady flow problem with the presence of the free surface. Hence Ogilvie's method, which depends on those two assumptions, may be considered equivalent to a time stepping method. Ogilvie solved the problem with an approximation of a thin body or thin bow. This is an extra simplification to the problem which is not necessary in the slender-ship theory. Daoud (1975) solved the same problem for the case of a bow with finite thickness. He satisfied the body boundary condition on the actual surface of the ship. Hence his method includes the diffraction of diverging waves by the body. The time-stepping method can be applied to problems involving lateral asymmetry. Chapman (1976) solved the problem for the case of a yawed slender surface-piercing flat plate.

However, since the time-stepping method reduces the problem to a two-dimensional unsteady flow problem, an initial condition at the bow is necessary. Ogilvie, Daoud,

and Chapman all assumed that there is no disturbance ahead of the bow or leading edge. This condition may be reasonable for a really slender body. However even for fairly slender bodies such as fine-ship models, it is observed that the wave elevation at the bow is of an appreciable magnitude (see Standing (1974) for example). Also, the acceleration becomes infinite at the leading edge according to the assumption of no disturbance ahead of the leading edge. In the bow near region, the inertia force is expected to dominate the gravitational force, but it is not infinite. Because of these reasons the effects of the disturbance ahead of the leading edge are expected to be significant even for a fairly slender body and by no means should be neglected in the strut problem. Furthermore, Ogilvie's method depends on the assumption of the slenderness of the body and so cannot be applied to a strut which is not slender.

Chapman showed that the nondimensional problem of the flow around a yawed surface-piercing flat plate can be determined by only two nondimensional parameters according to the time stepping method. One of them, named η , is the product of the Froude number and the square root of the aspect ratio, while the other, called $\mu\eta$, is the product of the Froude number and the ratio of angle of attack to the square root of the aspect ratio. On the other hand, it is known that the free-surface condition can be approximated by the pressure-relief surface condition at a very

high Froude number. Because of this reason it can be concluded that the flow around a yawed surface-piercing flat plate should be independent of Froude number and so independent of η or $\mu\eta$ at a high Froude number. Thus there are significant upstream effects of any disturbance, and so it seems that the assumption of the slenderness of the body, which leads to the time stepping method, is too strong. There is an essential difficulty in including effects of aspect ratio, at least at a high Froude number.

Ogilvie (1975) proposed an idea to overcome this difficulty. If the problem could be broken into two parts, one of which is a conventional infinite-fluid problem, the other a problem in wave propagation to which can be reasonably applied the time stepping method, the problem can be solved without great difficulty. The proposed method can be referred to as a hybrid method, since the problem will be solved partially three-dimensionally and partially two-dimensionally.

This idea of the hybrid method is developed and applied to the problem of the surface-piercing strut at a high Froude number. The side force acting on the strut and the wave pattern around the strut are calculated numerically for different Froude numbers and aspect ratios.

II. PROBLEM FORMULATION AND METHOD OF SOLUTION

Formulation of the problem

The strut is assumed to be fixed in an incident stream of velocity U. The coordinate system, as shown in Figure 1, has the Z-axis in the upward direction, the Y-axis positive to starboard and the X-axis parallel to and in the same direction as the incident stream. The origin of the coordinate system is set at the intersection of the trailing edge of the strut and the undisturbed free surface.

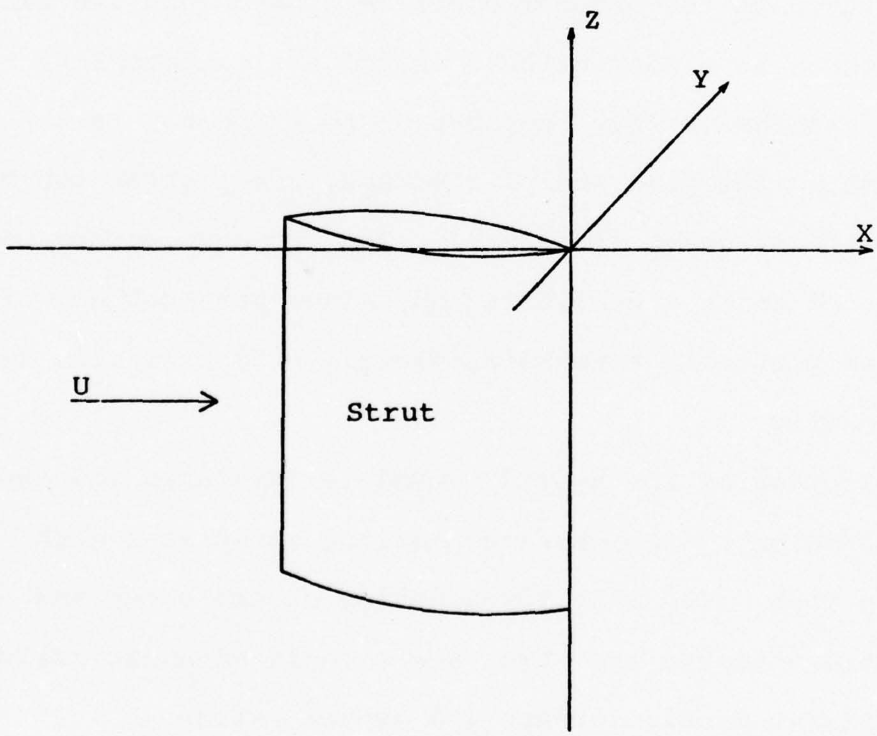

Figure 1. The coordinate system

The chord length and the depth of the strut are represented by L and D . Also H, SS, SP, SB, LE and TE will be used as abbreviated notations of the whole surface, the

starboard side surface, the port side surface, the bottom surface, the leading edge and the trailing edge of the strut respectively. The equation of the hull is given as:

$$H(X,Y,Z) = 0 \ .$$

Notation "H" will be used to denote either the surface of the strut or the function defining the surface of the strut.

The free surface is given as:

$$Z = \zeta_D(X,Y) \ .$$

The strut is assumed to be thin, with a small degree of lateral asymmetry. Also, the shape and dimensions of cross sections must vary slowly in the X direction.

In case of an asymmetric strut, a wake exists behind the strut. We assume that the wake occupies a thin sheet that joins the strut at its trailing edge and extends infinitely downstream, the surface of the sheet being given as

$$Y = W(X,Z) \ .$$

The usual assumptions are made about the fluid being inviscid and incompressible. The fluid motion is assumed to be irrotational except on the wake. There exists a velocity potential, which we write in the following form:

$$UX + \Phi(X,Y,Z) \ .$$

This velocity potential satisfies the Laplace equation in the fluid domain except at the wake:

$$\frac{\partial^2 \Phi}{\partial X^2} + \frac{\partial^2 \Phi}{\partial Y^2} + \frac{\partial^2 \Phi}{\partial Z^2} = 0 \quad ,$$

the dynamic free-surface boundary condition on the free surface:

$$g\zeta_D + \frac{1}{2}\{\Phi_X^2 + \Phi_Y^2 + \Phi_Z^2 + 2U\Phi_X\} = 0 \quad \text{on} \quad Z = \zeta_D(X,Y),$$

the kinematic free-surface boundary condition:

$$(U+\Phi_X)\zeta_{DX} + \Phi_Y\zeta_{DY} - \Phi_Z = 0 \quad \text{on} \quad Z = \zeta_D(X,Y),$$

and the rigid-surface boundary condition on the surface of the strut:

$$\Phi_n = -Un_X \quad \text{on } H,$$

where $n(X,Y,Z)$ is a unit normal vector, positive into the fluid domain. n_X , n_Y and n_Z are the components of $\underline{n}$ in X, Y, and Z directions. The dynamic condition on the wake, which states that there is no discontinuity of the pressure across the wake, is,

$$(2U\Phi_X + \Phi_X^2 + \Phi_Y^2 + \Phi_Z^2)\Big|_{Y=W+0} = (2U\Phi_X + \Phi_X^2 + \Phi_Y^2 + \Phi_Z^2)\Big|_{Y=W-0}, \tag{1}$$

and the kinematic condition which states that wake consists of stream lines is,

$$\Phi_Y - \{(U+\Phi_X)W_X + \Phi_Z W_Z\} = 0 \quad \text{on} \quad Y = W. \tag{2}$$

Finally, we must have a radiation condition which states that the wave propagation dies off faster upstream than downstream of the strut.

The method of solution

Step 1 The exact solution of the boundary-value problem as specified cannot be determined by existing

analytic techniques. Since our case includes asymmetric struts, we have to face the infinite perturbation velocity along the sharp leading edge as well as extra conditions at the wake in addition to the usual difficulties common to the problem of a symmetric body on the free surface. In particular, the location of the wake is not given explicitly. Also a discontinuity of the wave elevation is observed at the trailing edge. The precise investigation of the last problem, i.e., the discontinuity of the wave elevation at the trailing edge and the phenomena associated with it will be left for future works. However our theory should include the discontinuity of the wave level at the trailing edge and should be able to predict the fluid motion downstream of the trailing edge in some consistent way. This will be discussed later. Similarly the infinite perturbation velocity at the leading edge should be included in our theory.

The major difficulties left are the nonlinearity of the conditions at the free surface and the wake. As a first step toward making it possible to solve the problem, we shall approximate these nonlinear conditions with linear conditions. The free-surface conditions may be linearized by expanding the potential in a Taylor series about $Z=0$ and discarding all high order terms. Thus we obtain:

$$g\zeta_D + U\Phi_X = 0 \qquad \text{on} \quad Z=0$$

and

$$U\zeta_{D_X} = \Phi_Z \qquad \text{on} \quad Z=0$$

or, combining them together,

$$\Phi_{XX} + \kappa\Phi_Z = 0 \quad \text{on} \quad Z=0,$$

where

$$\kappa = g/U^2 \ .$$

The theoretical determination of the exact position of the wake is beyond our resources even in an infinite fluid. In this work, the wake is assumed to be a plane sheet parallel to the direction of the incident flow. We define S_w as the plane sheet that the wake occupies. Then S_w is given as:

$$Y = 0 \ , \quad X \geq 0 \ , \quad 0 \geq Z \geq -D \ .$$

We also define S_w^+ as the starboard side surface of S_w and S_w^- as the port side surface of S_w. These definitions are shown in Figure 2. This model of the wake violates the kinematic condition at the wake. The kinematic condition at the wake, (2), should be replaced by a weaker condition which states the continuity of the velocity in the normal direction across the wake. Let n_w be the normal vector on S_w^+ the surface of the wake, and $\Delta\Phi_{nw}$ be the difference of the velocity in the normal direction across the wake. Then the condition stated above can be given as

$$\Delta\Phi_{nw} = \Phi_{nw}\Big|_{S_w^+} - \Phi_{nw}\Big|_{S_w^-} = 0 \ .$$

The equation (1), the dynamic condition at the wake, becomes after linearization:

$$\Phi_X\Big|_{S_w^+} - \Phi_X\Big|_{S_w^-} = 0$$

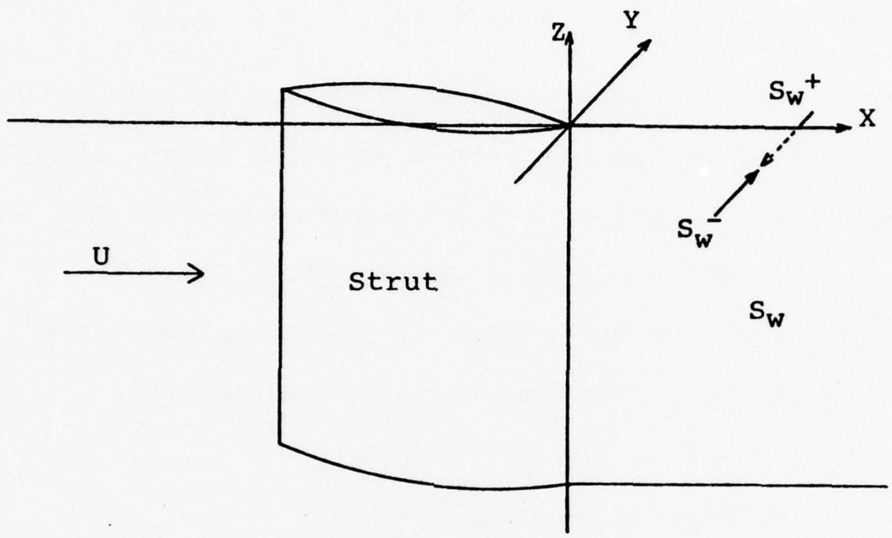

Figure 2. The wake model

Hence the potential jump across the wake

$$\Delta\Phi = \{\Phi\Big|_{S_w^+} - \Phi\Big|_{S_w^-}\}$$

is independent of X. Let $\Phi_s(Q)$ be the value of Φ on the surface of the strut. Also define $\Phi_{st}(Z)$ and $\Phi_{pt}(Z)$ as follows:

$$\lim_{Q \to TE \text{ on } SS} \Phi_s(Q) = \Phi_{st}(Z);$$

and

$$\lim_{Q \to TE \text{ on } SP} \Phi_s(Q) = \Phi_{pt}(Z) .$$

Using these notations, the condition obtained above can be stated as

$$\Delta\Phi(Z) = \Phi_{st}(Z) - \Phi_{pt}(Z) .$$

The value

$$\Phi_{st}(Z) - \Phi_{pt}(Z)$$

is the circulation around the strut, by definition; hence, finally,

$$\Delta\Phi(Z) = \Phi_{st}(Z) - \Phi_{pt}(Z) = \Gamma_D(Z) \qquad \text{on} \quad S_W \ .$$

For more discussion concerning the wake and the conditions at the wake we refer to Robinson & Laurmann (1956, 1.15).

The Laplace equation, the body boundary condition and the radiation condition remain unchanged.

It is convenient to rewrite the defining equations in nondimensional form. Nondimensional coordinates are:

$$x = X/L, \ y = Y/L, \text{ and } \ z = Z/L \ ,$$

and the nondimensional velocity potential is

$$\phi = \Phi/UL \ ;$$

also, the nondimensional wave height is

$$\zeta = \zeta_D/L \ .$$

Other nondimensional values are as following:

$$AR(= \text{aspect ratio}) = D/L$$

$$Fn(= \text{Froude number}) = U/\sqrt{gL}$$

$$\Gamma(z) = \Gamma_D(Z)/UL \ .$$

The unit normal vector, $\underline{n}(x,y,z)$, its components, n_x, n_y, n_z and the wake, S_W , and its surfaces, S_W^+ and S_W^- , are to be defined in nondimensional space. Then the nondimensional boundary-value problem is given as follows:
The Laplace equation in the fluid domain:

$$\phi_{xx} + \phi_{yy} + \phi_{zz} = 0, \tag{3}$$

the dynamic free surface condition:

$$\zeta + F_n^2\phi_x = 0 \quad \text{on} \quad z=0 \ , \tag{4}$$

and the kinematic free-surface condition:

$$\zeta_x = \phi_z \qquad \text{on} \quad z=0 \tag{5}$$

(or the combined linearized free-surface condition:

$$\phi_{xx} + F_n^{-2}\phi_z = 0 \quad \text{on} \quad z=0), \tag{6}$$

the body boundary condition:

$$\phi_n = -n_x \quad \text{on} \quad H, \tag{7}$$

the condition on the wake:

$$\Delta\phi_{nw} = \phi_{nw}\Big|_{S_w^+} - \phi_{nw}\Big|_{S_w^-} = 0 \qquad \text{on} \quad S_w \ , \tag{8}$$

$$\Delta\phi(z) = \phi_{st}(z) - \phi_{pt}(z) = \Gamma(z) \quad \text{on} \quad S_w \ . \tag{9}$$

Step 2 Although the boundary value problem we have to solve is linear now, it is still an extremely difficult one to solve. Therefore we will follow the idea of the hybrid method to further facilitate finding a solution of the problem.

Now we break the velocity potential ϕ into two parts as follows:

$$\phi = \phi_0 + \phi_1 \ .$$

Since the ϕ_0 portion of the problem is to be solved three-dimensionally, the free-surface condition should be simple. There are two choices. One of them is the condition

$$\phi_0 = 0 \qquad\qquad \text{on} \qquad z=0 \ ,$$

and the other is the condition

$$\phi_{0_z} = 0 \qquad \text{on} \quad z=0 \ .$$

To decide which condition to choose, we refer to the beautiful pictures issued by NSRDC (Rothblum, Mayer and Daily, 1972) of yawed sharp-edged surface-piercing struts moving at high speed. In many of those pictures, cavitation is observed along the leading edge on the suction side. This coincides with the assumed infinite perturbation velocity along the leading edge. However it is clearly shown in those pictures that the cavitation region is clearly separated from the free surface. Also the free-surface elevation near the leading edge is quite different from what would be obtained by assuming that the flow on the free surface rounds the sharp leading edge. Therefore we propose a condition stating that the flow on the free surface does not round the sharp leading edge. To be consistent with this condition, we must use the former free-surface condition,

$$\phi_0 = 0 \qquad \text{on} \quad z=0 \ , \tag{10}$$

instead of the rigid-wall free-surface condition.

On the surface of the strut we use the condition:

$$\phi_{0_n} = -n_x \qquad \text{on} \quad H, \tag{11}$$

for the ϕ_0 portion of the velocity potential. Other conditions which should be satisfied by the ϕ_0 portion of the velocity potential are the Laplace equation in the fluid domain (except at the wake),

$$\phi_{0_{xx}} + \phi_{0_{yy}} + \phi_{0_{zz}} = 0 \;, \tag{12}$$

the conditions at the wake,

$$\Delta\phi_{0_{nw}} = 0 \qquad \text{on} \quad S_w \;, \tag{13}$$

$$\Delta\phi_0(z) = \Gamma_0(z) \qquad \text{on} \quad S_w \;. \tag{14}$$

Also, a condition,

$$\lim_{x\to-\infty} \phi_0 = 0 \;, \tag{15}$$

is assumed, so that ϕ_0 can be determined uniquely.

Because the ϕ_1 portion of the velocity potential is defined by

$$\phi_1 = \phi - \phi_0 \;,$$

it should satisfy the Laplace equation in the fluid domain (except at the wake):

$$\phi_{1_{xx}} + \phi_{1_{yy}} + \phi_{1_{zz}} = 0 \;, \tag{16}$$

the linearized free-surface condition:

$$\phi_{1_{xx}} + F_n^{-2}\phi_{1_z} + F_n^{-2}\phi_{0_z} = 0 \qquad \text{on} \quad z=0 \tag{17}$$

(or, separated into the dynamic and the kinematic free-surface conditions,

$$\zeta + F_n^{2}\phi_{1_x} = 0 \qquad \text{on} \quad z=0, \tag{18}$$

$$\zeta_x = \phi_{0_z} + \phi_{1_z} \qquad \text{on} \quad z=0), \tag{19}$$

the condition on the surface of the strut:

$$\phi_{1_n} = 0 \qquad \text{on} \quad H,$$

and the conditions at the wake:

$$\Delta\phi_{1_{nw}} = 0 \qquad \text{on} \quad S_w,$$

$$\Delta\phi_1(z) = \Gamma(z) - \Gamma_0(z) = \Gamma_1(z) \qquad \text{on} \quad S_w.$$

A radiation condition is also necessary. However, since the time stepping method will be used eventually, and hence the waves will propagate downstream only, it is not necessary to define the radiation condition in terms of ϕ_0 and ϕ_1 . The condition for the uniqueness of the solution will suffice for a condition at infinity.

It is observed that at a high Froude number the first crest of the transverse waves lies far behind the strut and the diverging waves are dominant around and close behind the strut. The crests of these diverging waves are almost parallel to the incident stream (or the x-axis); hence it is reasonable to assume that

$$\frac{\partial}{\partial x} = o\left(\frac{\partial}{\partial y}, \frac{\partial}{\partial z}\right)$$

for the ϕ_1 portion of the problem. This is because these waves motions are represented by the ϕ_1 portion of the velocity potential. Hence the ϕ_1 portion of the problem can be reasonably approximated by a sequence of two-dimensional problems in the transverse planes, the potentials for which satisfy the two-dimensional Laplace equation in the fluid domain, except at S_w ,

$$\phi_{1_{yy}} + \phi_{1_{zz}} = 0 , \tag{20}$$

the linearized free surface condition

$$\phi_{1_{xx}} + F_n^{-2}\phi_{1_z} + F_n^{-2}\phi_{0_z} = 0 \qquad \text{on} \quad z=0 \tag{21}$$

(or, separated,

$$\zeta + F_n{}^2\phi_{1_x} = 0 \qquad \text{on} \quad z=0, \tag{22}$$

and

$$\zeta_x = \phi_{1_z} + \phi_{0_z} \qquad \text{on} \quad z=0), \tag{23}$$

the conditions at the wake:

$$\Delta\phi_{1_{nw}} = 0 \qquad \text{on} \quad S_w \,, \tag{24}$$

$$\Delta\phi_1(x) = \Gamma_1(z) \qquad \text{on} \quad S_w \,. \tag{25}$$

The body boundary condition on the surface of the strut is to be satisfied approximately in the two dimensions of the transverse planes as:

$$\phi_{1_N} = 0 \qquad \text{on} \quad H \,, \tag{26}$$

where $\underline{N}$ is the two-dimensional normal. A condition stating that

$$\phi_1 \to 0 \qquad \text{as} \quad x \to -\infty \qquad \text{or} \quad \sqrt{y^2+z^2} \to \infty \tag{27}$$

is assumed in order to make the solution unique.

Equation (17), the free surface condition for ϕ_1, shows that ϕ_1 is the velocity potential for a fluid motion which could be caused by an applied pressure distribution on the free surface. If we let the equivalent pressure distribution, nondimensionalized by ρU^2, be represented by $P_0(x,y)$, its form is given by:

$$P_{0_x}(x,y) = F_n^{-2}\phi_{0_z} \,.$$

The disturbance caused by a concentrated pressure distribution on a stream is known to die off much more quickly upstream than downstream (see, for example, Peters 1949).

Because of this reason, it seems reasonable to assume that the ϕ_1 portion of the problem is independent of the condition downstream. This condition can be understood more clearly by transferring the x-axis into a nondimensional time domain. Then the ϕ_1 portion of the problem can be formulated as a two-dimensional unsteady-flow problem. Especially behind the trailing edge where there is no rigid boundary, the ϕ_1 problem can be considered as a generalized two-dimensional Cauchy-Poisson problem. Hence the solution of the ϕ_1 problem in the region behind the trailing edge can be obtained just by evaluating a double integral (for example, see Lamb, 1932). The notation x and t will be freely used interchangeably for convenience hereafter.

The ϕ_0 portion of the problem corresponds to the case of infinite Froude number and so includes no effect of gravity. In the following chapters, the ϕ_0 portion of the problem will be mentioned as the "zero-gravity problem" or simply as the "ϕ_0 problem". The ϕ_1 portion of the problem includes all the wave motion and hence will be mentioned as the "wave problem" or as the "ϕ_1 problem".

III. SOLUTION OF THE ZERO-GRAVITY PROBLEM (ϕ_0 PROBLEM)

The problem of the ϕ_0 portion of the velocity potential can be looked at as a three-dimensional problem of a body in an infinite fluid. Methods originally developed for a lifting or nonlifting body problem in an infinite fluid (Hess & Smith, 1966; Hess, 1972; Webster, 1975; etc.) can be modified so that they can be applied to the ϕ_0 problem.

However, a rather basic method is to be used in this work. Define a control surface S_t and a coordinate system $O\xi\eta\rho$ as in Figure 3, where

$$S_t = H + S_f + S_r + S_w^+ + S_w^- \; . \tag{28}$$

S_w^+ and S_w^- were previously defined as the starboard- and port- side surface of the wake, extending to infinity downstream. However, S_w^+ and S_w^- are now redefined as the parts of the original S_w^+ or S_w^- bounded by S_r and H. S_r is the part of the hemispherical surface defined by:

$$\rho \leq 0 \; , \; \xi^2 + \eta^2 + \rho^2 = R^2$$

and bounded by S_w^+ and S_w^-. H is the surface of the strut except for the trailing edge. S_f is the part of the undisturbed free surface bounded by H, S_r , S_w^+ and S_w^- . Also, V is the fluid domain bounded by the closed control surface S_t .

According to Green's formula, ϕ_0 at a point P(x,y,z) in V can be written in terms of ϕ_0 and ϕ_{0_n} over S_t

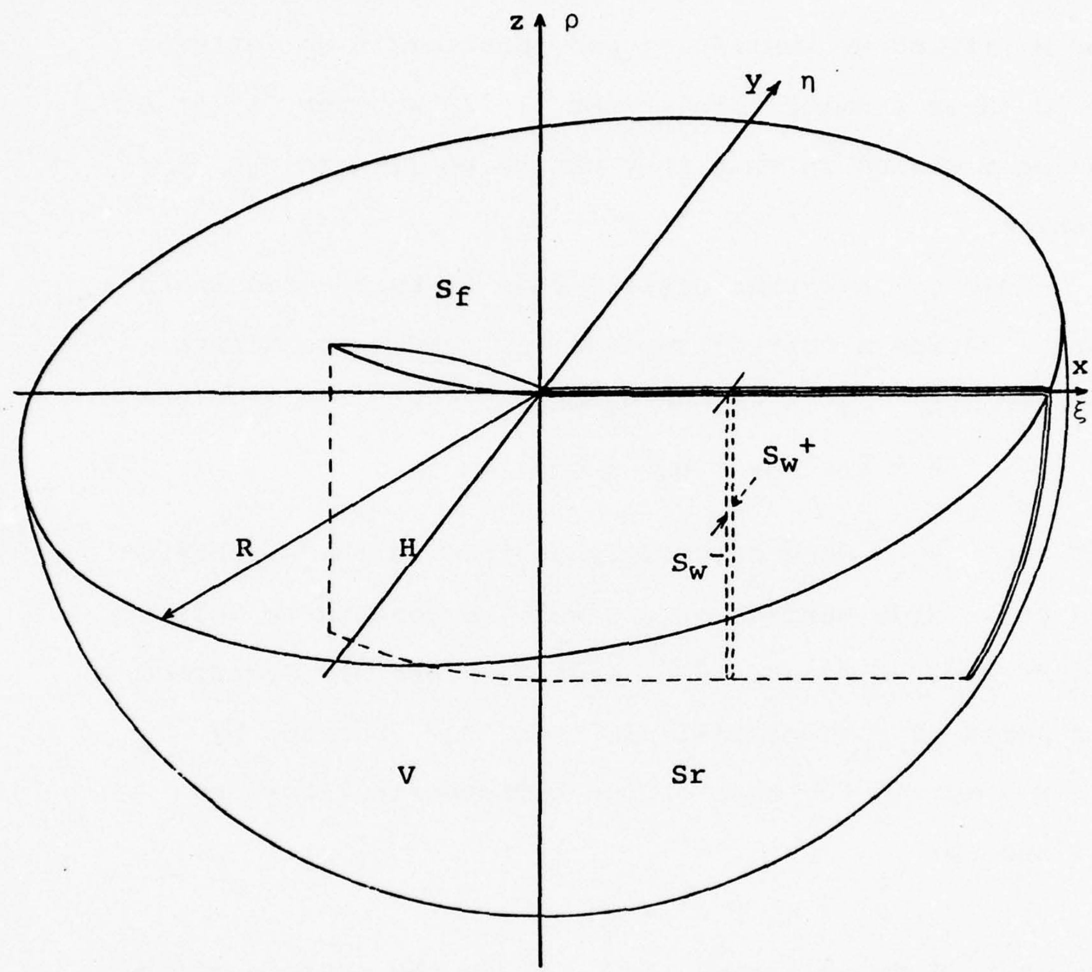

Figure 3. The Coordinate System and the Control Surface

as follows:

$$\phi_0(P) = \frac{1}{4\pi}\iint_{St}\{\phi_0(Q)\cdot G_n(P;Q) - \phi_{0_n}(Q)\cdot G(P;Q)\}ds(Q) \ , \tag{29}$$

where $\underline{n}$ is positive into the fluid. Choose $G(P;Q)$ of the following form:

$$G(P;Q) = \frac{1}{r(P;Q)} - \frac{1}{r'(P;Q)} \ , \tag{30}$$

where P is the point (x,y,z) and Q is (ξ,η,ρ), and

$$\begin{aligned} r(P;Q) &= \{(x-\xi)^2+(y-\eta)^2+(z-\rho)^2\}^{1/2} \\ r'(P;Q) &= \{(x-\xi)^2+(y-\eta)^2+(z+\rho)^2\}^{1/2} \end{aligned} \tag{31}$$

Then $G = 0$ on the undisturbed free surface, and because of the free-surface condition for ϕ_0 ,

$$\phi_0 = 0 \quad \text{on} \quad z = 0 \ .$$

The contribution of the integral over S_f in Equation (29) is zero. Also the contribution of the integral over S_r is known to vanish as $R \to \infty$, from condition (15), which states that there is no disturbance far upstream of the strut. Since $\underline{n}$ on S_w^+ , which we shall denote by $\underline{n}_w$, equals $-(\underline{n})$ on S_w^- , the integral over S_w^+ and S_w^- can be combined as an integral over S_w as follows:

$$\frac{1}{4\pi}\iint_{S_w^+}(\phi_0 G_n - \phi_{0_n} G)dS + \frac{1}{4\pi}\iint_{S_w^-}(\phi_0 G_n - \phi_{0_n} G)dS$$

$$= \frac{1}{4\pi}\iint_{S_w^+}(\phi_0 G_{nw} - \phi_{0_{nw}} G)dS + \frac{1}{4\pi}\iint_{S_w^-}(-\phi_0 G_{nw} + \phi_{0_{nw}} G)dS$$

$$= \frac{1}{4\pi} \iint\limits_{S_w} \left\{ \left[\phi_0 \Big|_{S_w^+} - \phi_0 \Big|_{S_w^-} \right] G_{n_w} \right.$$

$$\left. - \left[\phi_{0_{nw}} \Big|_{S_w^+} - \phi_{0_{nw}} \Big|_{S_w^-} \right] G \right\} dS$$

$$= \frac{1}{4\pi} \iint\limits_{S_w} \{ \Delta\phi_0 \, G_{n_w} - \Delta\phi_{n_w} \, G \} \, dS \, .$$

Because of the shape of S_w and the conditions (13) and (14), the above integral can be reduced to

$$\frac{1}{4\pi} \int_{-AR}^{0} d\rho \Gamma_0(\rho) \cdot \int_0^R G_\eta \, d\xi \, .$$

Finally, after making R infinite, equation (29) is reduced to

$$\phi_0(P) = \frac{1}{4\pi} \iint\limits_H \{ \phi_0(Q) \, G_n - \phi_{0_n}(Q) \, G \} \, dS$$

$$+ \frac{1}{4\pi} \int_{-AR}^{0} d\rho \Gamma_0(\rho) \int_0^\infty G_\eta \, d\xi \, . \tag{32}$$

Because ϕ_{0_n} over H is known from the body boundary condition and $\Gamma_0(\rho)$ is given as the potential jump at the trailing edge, the only unknown in the right-hand side of (32) is ϕ_0 on H , the surface of the strut. Now take the limit as P approaches the point Q_s on the surface of the strut:

$$\lim_{P \to Q_s} \phi_0(P) = \frac{1}{2}\phi_0(Q_s) + \frac{1}{4\pi} \iint\limits_H \{ \phi_0(Q) G_n - \phi_{0_n}(Q) G \} dS$$

$$+ \frac{1}{4\pi} \int_{-AR}^{0} d\rho \Gamma_0(\rho) \int_{0}^{\infty} G_\eta \, d\xi \; .$$

The bar through the integral signs denotes that a principal value interpretation should be made. Finally

$$\phi_0(Q_s) = \frac{1}{2\pi} \iint_H \{\phi_0(Q) G_n - \phi_{0_n}(Q) G\} \, dS$$

$$+ \frac{1}{2\pi} \int_{-AR}^{0} d\rho \Gamma_0(\rho) \int_{0}^{\infty} G_\eta \, d\xi \; ; \tag{33}$$

this is an integral equation to be solved.

Equation (33) is a Fredholm integral equation of the second kind, which can be solved numerically without difficulty. Then, using ϕ_0 on the surface of the strut in the equation (32), the velocity potential ϕ_0 anywhere in V can be obtained. Differentiating both sides of (32) with respect to z, we obtain the equation:

$$\phi_{0_z}(P) = \frac{1}{4\pi} \iint_H \{\phi_0(Q) \, G_{nz} - \phi_{0_n} \, G_z\} \, dS$$

$$+ \frac{1}{4\pi} \int_{-AR}^{0} d\rho \Gamma_0(\rho) \int_{0}^{\infty} G_{\eta z} \, d\xi \; . \tag{34}$$

Thus we can obtain the value of ϕ_{0_z} on the free surface, which is necessary to solve ϕ_1 problem. Also, since the ϕ_0 distribution on the surface of the strut is known, we can calculate the pressure distribution and finally the forces acting on the strut due to the ϕ_0 portion of the problem.

IV. THE WAVE PROBLEM (ϕ_1 problem)

In chapter II, the ϕ_1 portion of the problem was formulated as a two-dimensional unsteady-flow problem. To solve this problem, we use a method which is essentially equivalent to what Chapman (1976) used to solve his problem. This method, roughly speaking, consists of two iterated steps. One of the steps is to obtain enough information to solve the two-dimensional boundary-value problem at a certain time (station) from the solution at the previous time (station). The other one is the solution of the two-dimensional boundary-value problem at each time or station.

Consider a problem at $t = t_n$ or at a corresponding transverse plane given as $x = x_n$. Let CF, CH and CR be the contours shown in Figure 4 and CT be the closed control contour given as:

$$CT = CH + CF + CR ;$$

also let S be the region bounded by CT. Using Green's formula, ϕ_1 at a point $P(y,z)$ in S can be written in terms of ϕ_1 and ϕ_{1_N} on CT as follows:

$$\phi_1(P) = \frac{1}{2\pi} \int_{CT} \{\phi_1(Q)\ G_N(P;Q) - \phi_{1_N}(Q)\ G(P;Q)\}\ d\ell. \quad (35)$$

We choose the two-dimensional Green function, $G(P;Q)$, as:

$$G = \log \frac{r'(P;Q)}{r(P;Q)} ;$$

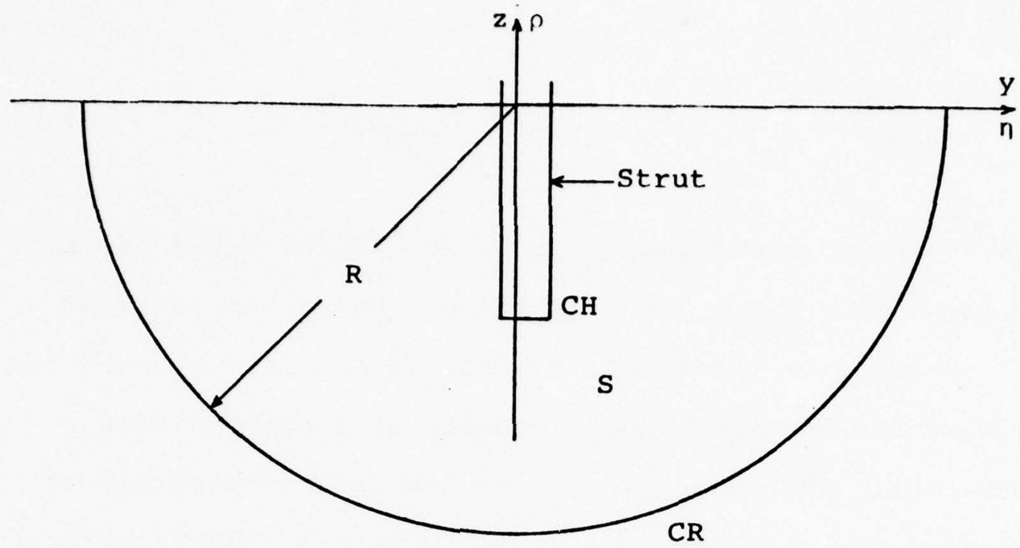

Figure 4. Contours in ϕ_1 problem

where P is the point $(y;z)$ and Q is $(\eta;\rho)$ and

$$r = \{(y-\eta)^2+(z-\rho)^2\}^{1/2},\ r' = \{(y-\eta)^2+(z+\rho)^2\}^{1/2}.$$

Then, for Q on the free surface,

$$G(P;Q) = 0.$$

We also have conditions for ϕ_1:

$$\phi_{1_N} = 0 \qquad \text{on} \quad CH,$$

and

$$\lim_{R\to\infty} \phi_1 = 0 \qquad \text{on} \quad CR .$$

When these conditions are applied, Equation (35) is reduced to:

$$\phi_1(P) = \frac{1}{2\pi}\int_{CF} \phi_1(Q)\ G_N(P;Q)\,d\ell$$

$$+ \frac{1}{2\pi}\int_{CH} \phi_1(Q)\ G_N(P;Q)\,d\ell\ . \qquad (36)$$

Assume $Q_S(x,y)$ is a point on CH and take a limit as P approaches Q_S . Then Equation (36) becomes an integral

equation,

$$\phi_1(Q_s) = \frac{1}{\pi}\int_{CF} \phi_1 G_N d\ell + \frac{1}{\pi}\fint_{CH} \phi_1 G_N d\ell \ , \tag{37}$$

which can be solved for ϕ_1 on CH if the value of ϕ_1 on the free surface, CF , is known. Hence the value of ϕ_1 on the free surface is all the information we need for solving the two-dimensional problem at a certain time (station). The value of ϕ_1 on the free surface can be obtained from the solution at the previous time (station). This can be performed through the free-surface conditions. The linearized dynamic free-surface condition is

$$\zeta + F_n^2 \phi_{1_t} = 0;$$

and the kinematic condition is

$$\zeta_t = \phi_{1_z} + \phi_{0_z} \ .$$

We may use these conditions to obtain the value of ϕ_1 on the free surface at $t = t_n$ in the following way:

$$\begin{aligned}\phi_1\Big|_{t=t_n} &= \phi_1\Big|_{t=t_n-\Delta t_n} + \Delta t_n\, \phi_{1_t}\Big|_{t=t_n-\Delta t_n/2} \\ &= \phi_1\Big|_{t=t_n-\Delta t_n} - F_n^{-2}\Delta t_n \zeta\Big|_{t=t_n-\Delta t_n/2},\end{aligned}$$

where:

$$\Delta t_n = t_n - t_{n-1} \ .$$

This is because $\phi_{1_t} = -\zeta F_n^{-2}$. Also the value of ζ at $t=t_n-\Delta t_n/2$ can be obtained as follows:

$$\zeta\Big|_{t=t_n-\Delta t_n/2} = \zeta\Big|_{t=t_n-\Delta t_n-\Delta t_{n-1}/2}$$

$$+ (1/2)(\Delta t_n+\Delta t_{n-1})\zeta_t\Big|_{t=t_{n-1}} .$$

Since $\zeta_t = \phi_{1_z} + \phi_{0_z}$, we have

$$\zeta\Big|_{t=t_n-\Delta t_n/2} = \zeta\Big|_{t=t_n-\Delta t_n-\Delta t_{n-1}/2}$$

$$+ (1/2)(\Delta t_n+\Delta t_{n-1})(\phi_{1_z}+\phi_{0_z})\Big|_{t=t_{n-1}} . \tag{38}$$

Finally we obtain:

$$\phi_1\Big|_{t=t_n} = \phi_1\Big|_{t=t_n-\Delta t_n} - F_n^{-2}\Delta t_n\Big(\zeta\Big|_{t=t_n-\Delta t_n-\Delta t_{n-1}/2}$$

$$+ (1/2)(\Delta t_n+\Delta t_{n-1})(\phi_{1_z}+\phi_{0_z})\Big|_{t=t_{n-1}}\Big) . \tag{39}$$

Thus the value of ϕ_1 on the free surface and the wave height are obtained in terms of the solution at the previous time or at the transverse section ahead of the present section. The accuracy of approximation can be made to any required degree by making Δt smaller.[+]

Now the value of ϕ_1 on the free surface is known. Then the integral equation, (37), which is a Fredholm integral equation of the second kind, can be solved and the value of ϕ_1 on CH, the surface of the strut, can be obtained. We may differentiate both sides of Equation (36), to obtain the equation:

[+]Chapman (1976) showed that this procedure is stable numerically.

$$\phi_{1_z}(P) = \frac{1}{2\pi}\int_{CF} \phi_1 G_{N_z}(P;Q)\,d\ell$$

$$+ \frac{1}{2\pi}\int_{CH} \phi_1 G_{N_z}(P;Q)\,d\ell \ . \qquad (40)$$

Since we have the value of ϕ_1 on CF and CH, the value of ϕ_{1_z} on the free surface can be calculated using Equation (40), and then it will be used in calculating the value of ϕ_1 on the free surface at the next time station.

We need an initial condition to start the time-stepping procedure described above. The initial condition is given as the condition of no disturbance far ahead of the leading edge which can be stated as:

$$\phi_1 = 0 \quad \text{and} \quad \zeta = 0 \quad \text{at} \quad t = t_1 = -T;$$

where

$$T >> 1.$$

This condition includes the condition:

$$\phi_{1_z} = 0 \ \text{ on the free surface at } \ t = t_1.$$

The time-stepping procedure can be started in the following way using these initial conditions. At $t = t_1$, $\phi_{1_z} = 0$; hence

$$\zeta_t\Big|_{t=t_1} = \phi_{0_z}\Big|_{t=t_1} .$$

Since $\zeta = 0$ at $t = t_1$,

$$\zeta\Big|_{t=t_1+\Delta t_2/2} = (1/2)\Delta t_2 \zeta_t\Big|_{t=t_1}$$

$$= (1/2)\Delta t_2 \phi_{0_z}\Big|_{t=t_1} .$$

Because of the dynamic free surface condition

$$\phi_{1_t} = -F_n^{-2}\zeta \ ;$$

and since $\phi_1 = 0$ at $t = t_1$:

$$\phi_1\Big|_{t=t_2} = -F_n^{-2}(1/2)\Delta t_2 \phi_{0_z}\Big|_{t=t_2} .$$

Now that the value of ϕ_1 at $t = t_2$ and the value of ζ at $t = t_1+(1/2)\Delta t_2$ are obtained, we can use the usual time-stepping procedure given by the equations (38) and (39), and we can go on. It must be noticed that in the region ahead of the leading edge, it is not necessary to solve the integral equation. Since in this region the contour CH does not exist, we can use the equation (40) directly to calculate ϕ_{1_z} on the free surface.

Just behind the trailing edge, the wave elevation is not continuous. The two-dimensional unsteady fluid motion started from a discontinuous wave elevation is known to be highly singular at the point where the discontinuity of the wave elevation was originally located (see Lamb, 1932, for example). Therefore the solution is expected to be singular along the x-axis behind the trailing edge. However, for numerical solution, we can approximate the discontinuous wave elevation by a steep but continuous wave elevation. The time-stepping procedure proposed can be applied for this approximated wave elevation at the trailing edge and the results should be reasonable for phenomena the scale of which is not too small.

V. NUMERICAL RESULTS AND CONCLUSIONS

A computer program was developed based on the analysis given in the previous chapters to solve the problem numerically. Figure 5 shows in plan view the strut for which the numerical calculation performed, and Figure 6 shows the strut orientation. The results were obtained for three cases of aspect ratios, namely 0.5, 1 and 3, at different Froude numbers. The angle of attack was $\tan^{-1} 0.08$ ($\approx 5°$).

Some examples of the results of intermediate steps or the results of ϕ_0 problem are shown in Figures 7 - Figure 9 . In Figure 7 is shown a typical spanwise distribution of side force in nondimensional form due to the ϕ_0 portion of the potential. The side-force coefficient is plotted vs. aspect ratio in Figure 8. Also a typical example of the distribution of ϕ_{0_z} over the free surface is shown in Figure 9 . It should be mentioned that the corresponding fictitious-pressure distribution over the free surface which excites the ϕ_1 portion of the problem is finite and smooth everywhere on the free surface.

Final results are shown in Figures 10 - 16. The total side-force coefficient for the case of 0.5 aspect ratio plotted in Figure 10 shows good agreement with the experimental result for a yawed surface-piercing flat plate reported by Van Den Brug (1971), provided Froude number is not too small.

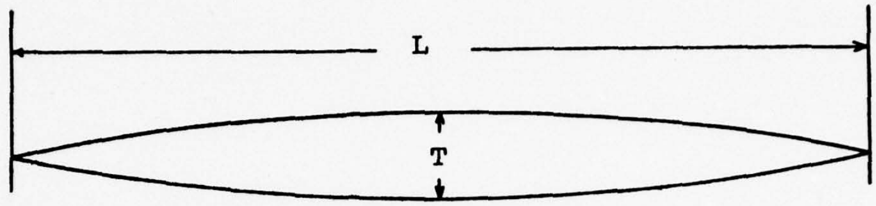

Figure 5. The strut in plan view

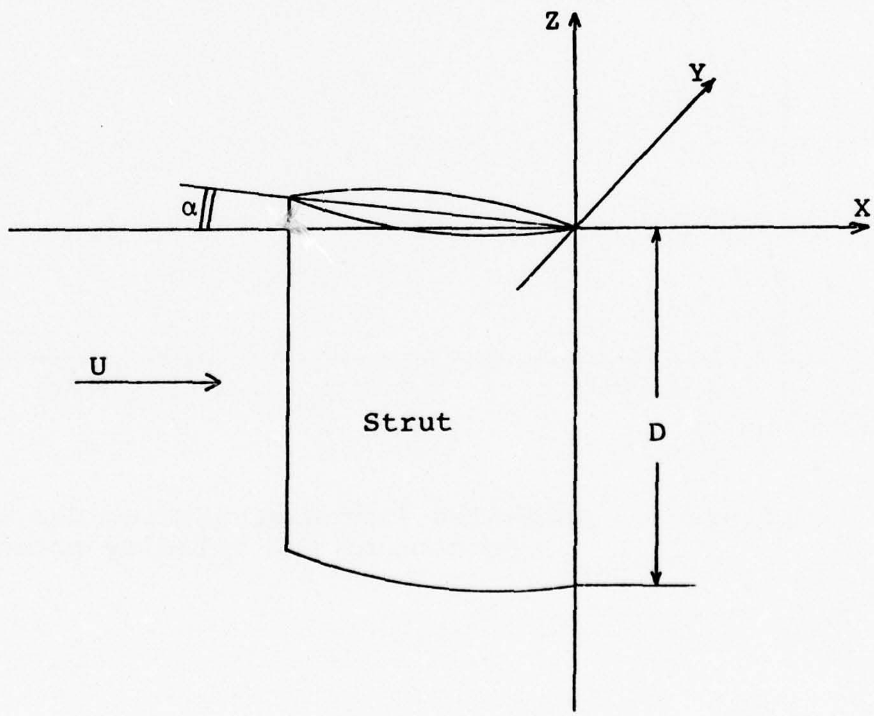

Figure 6. The strut in the incident stream

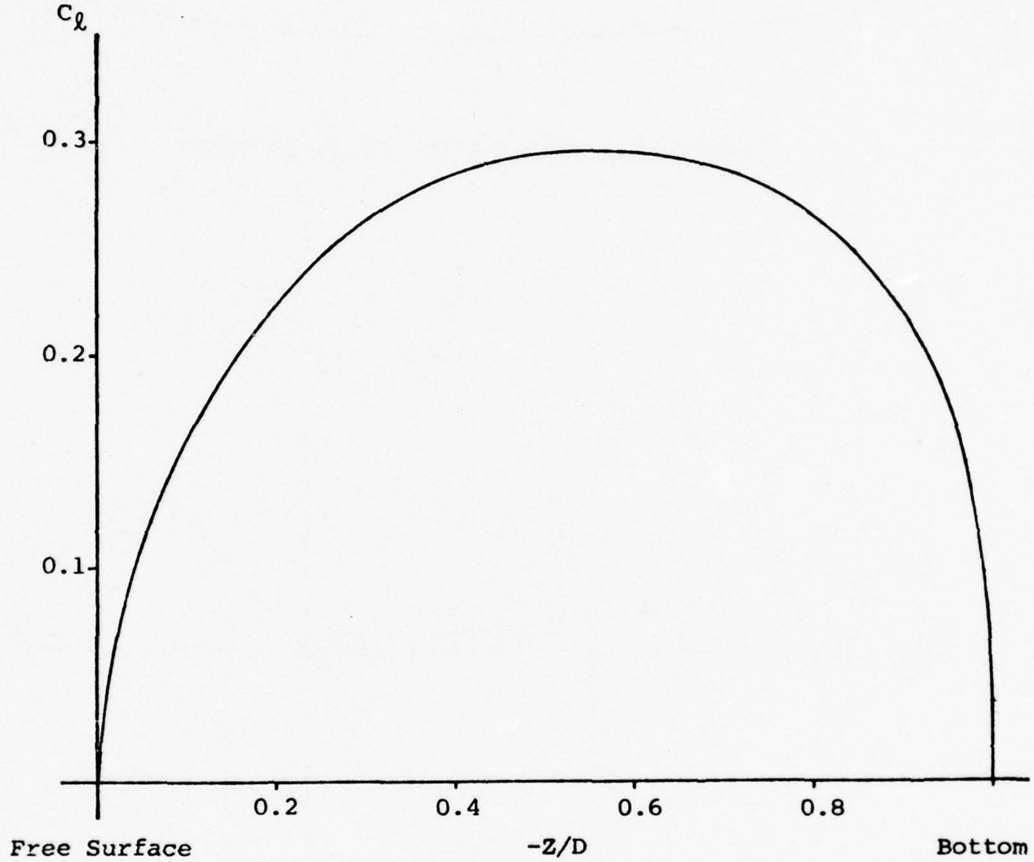

Figure 7. Span-wise lift distribution due to the ϕ_0 portion of the velocity potential

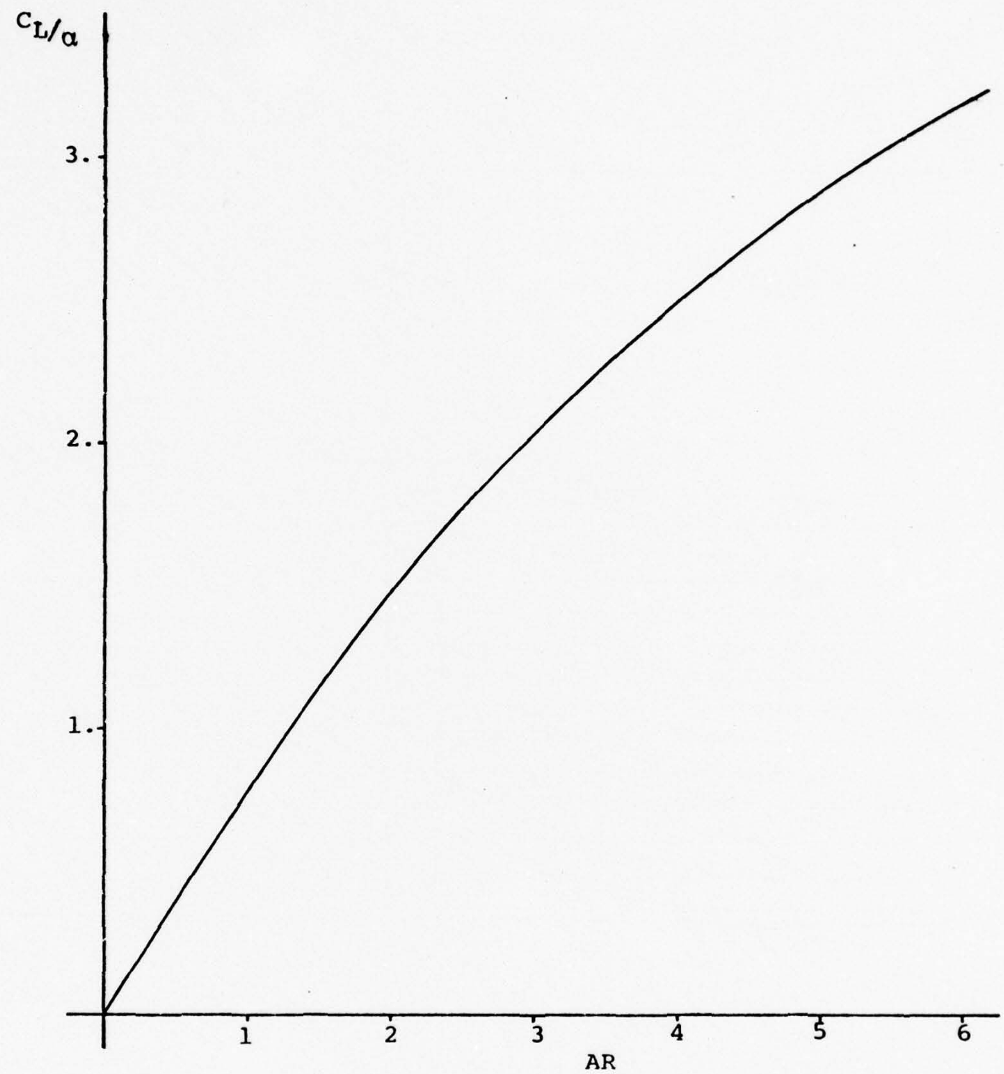

Figure 8. C_L/α, effect of aspect ratio

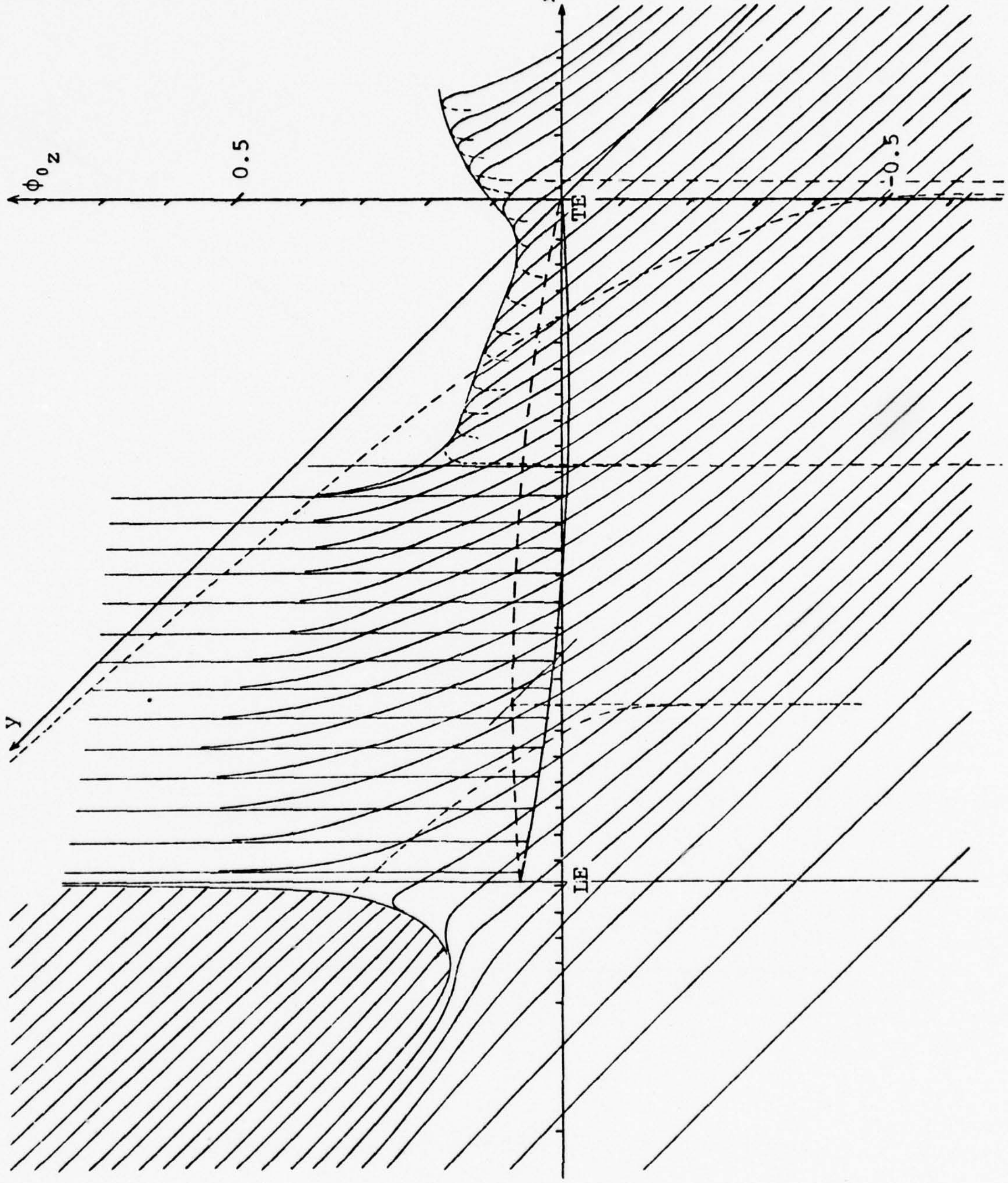

Figure 9. ϕ_{0_z} distribution on the free surface

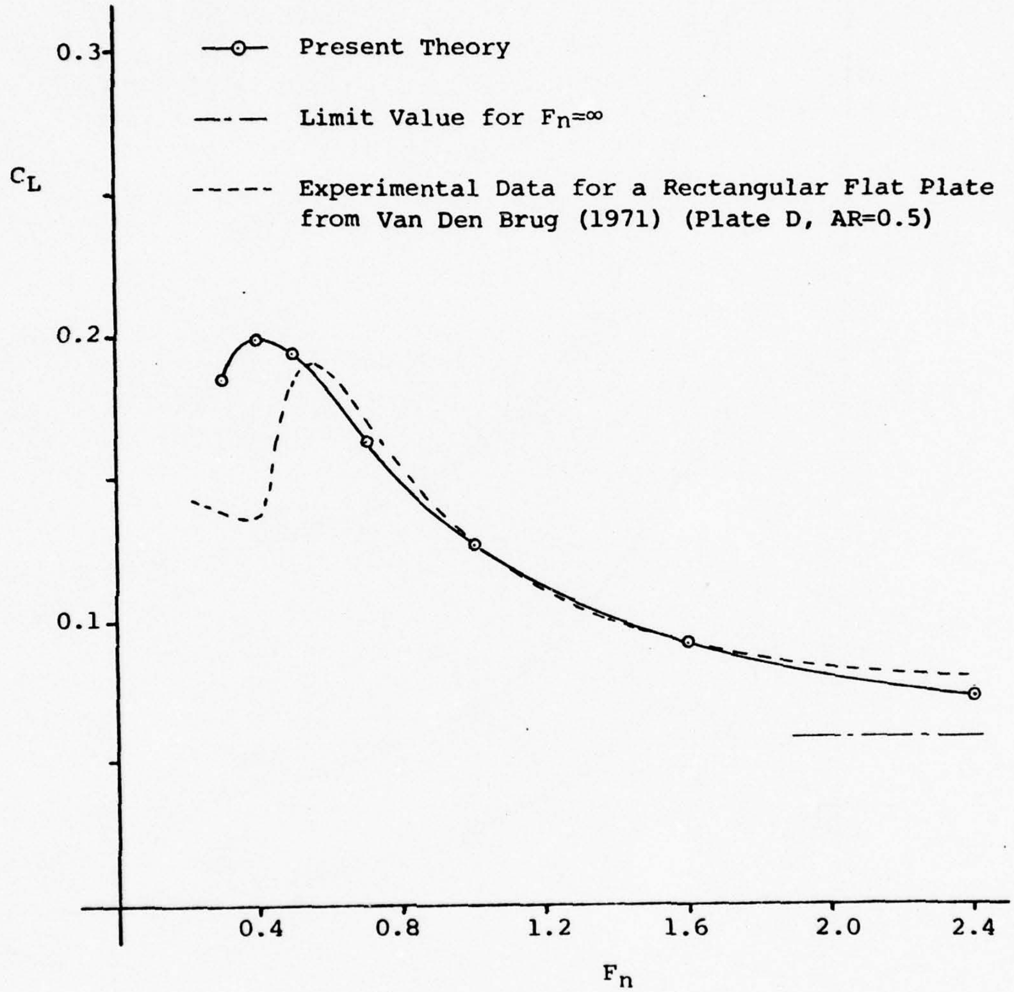

Figure 10. Side-force coefficient, $\tan \alpha = 0.08$, AR = 0.5

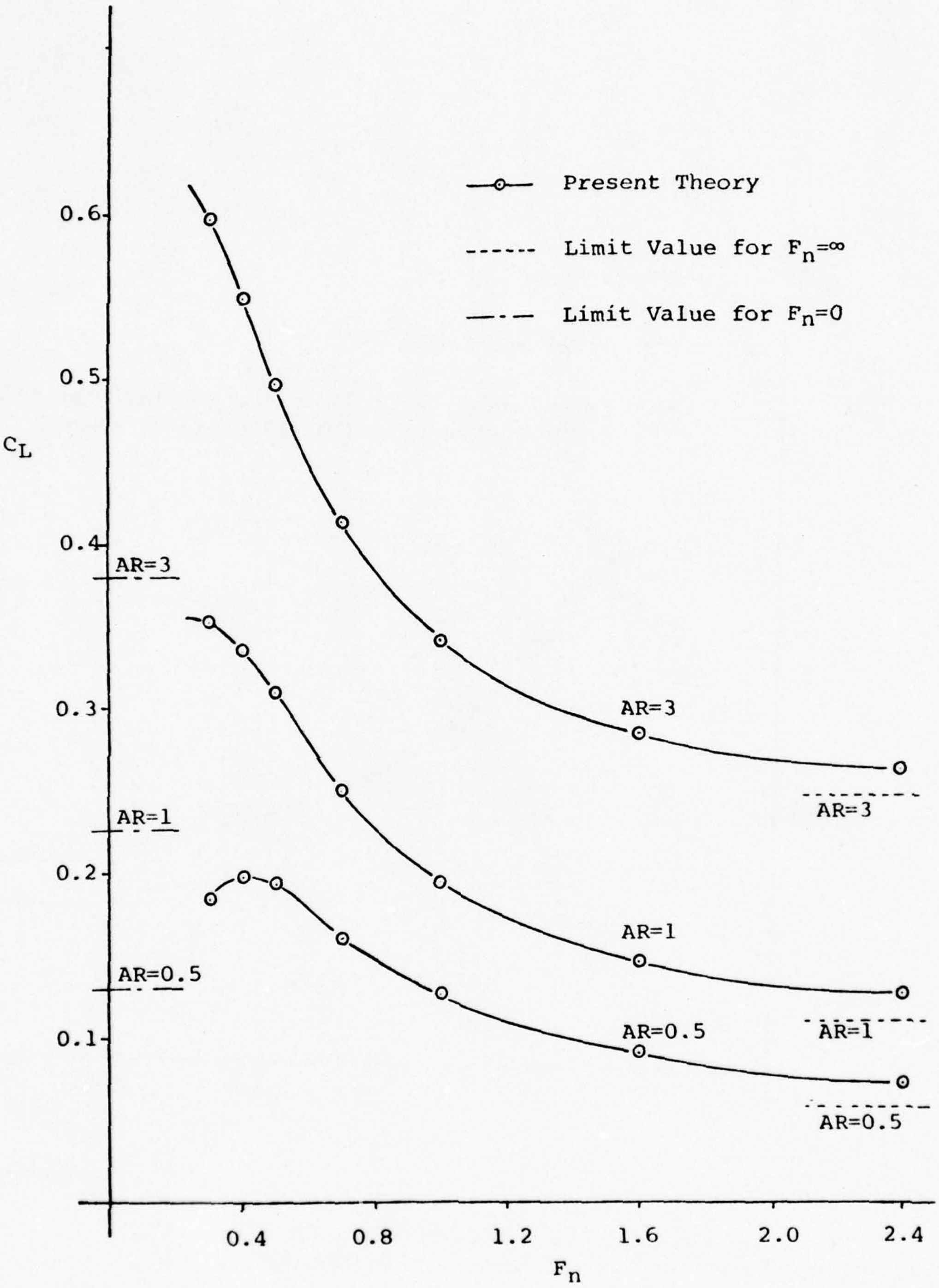

Figure 11. Side-force coefficient, effect of aspect ratio and Froude number $\tan \alpha = 0.08$, AR = 0.5, 1 and 3

The present theory does not involve the assumption of small aspect ratio. In Figure 11, the total side-force coefficients are plotted vs. Froude number for different aspect ratios. For infinite Froude number, the linearized free-surface condition is

$$\phi = 0 \qquad \text{on} \qquad z = 0.$$

This is the case of the ϕ_0 problem; hence it is reasonable that the total side-force coefficients are approaching the values from the ϕ_0 problems for corresponding aspect ratio as the Froude number increases. As the Froude number decreases from infinity, the total side-force coefficients increase to the maximum value near Froude number equal 0.4 . It should be noticed that these peak values are much bigger than the values from the infinite-Froude-number problems. This means that the effects of the waves on the side-force cannot be neglected unless the Froude number is extremely high.

For Froude-number equal zero, the linearized free surface condition reduces to

$$\phi_z = 0 \qquad \text{on} \qquad z = 0 .$$

This is the case of the double-body problem. The results from the double-body problems for corresponding aspect ratios are also shown in Figure 11. However, the present theory cannot be expected to converge to these values as the Froude number becomes zero, since an explicit assumption of high Froude number was made in formulating and solving the problem. In Figures 12 - 16 are shown the wave patterns

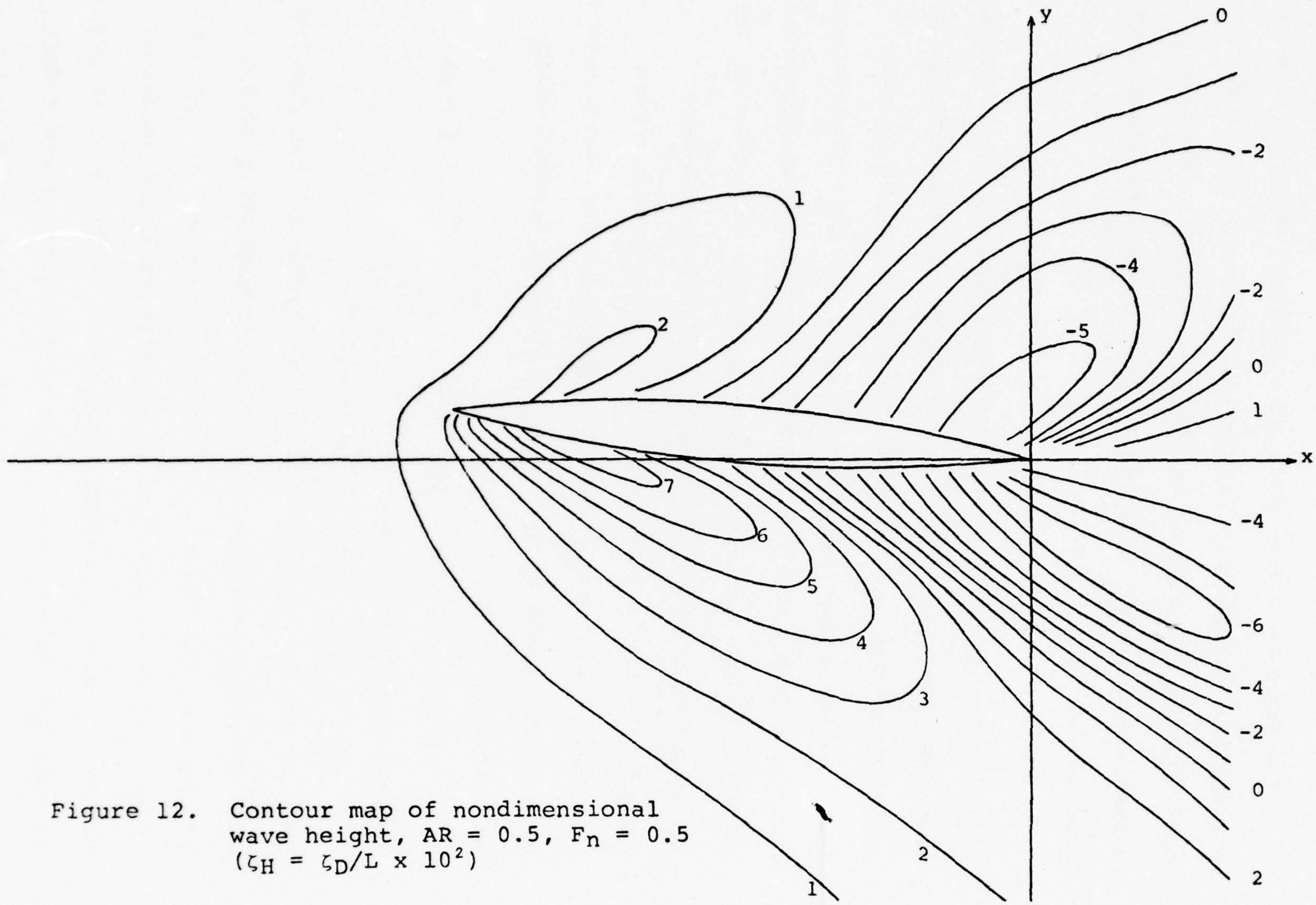

Figure 12. Contour map of nondimensional wave height, AR = 0.5, F_n = 0.5 ($\zeta_H = \zeta_D/L \times 10^2$)

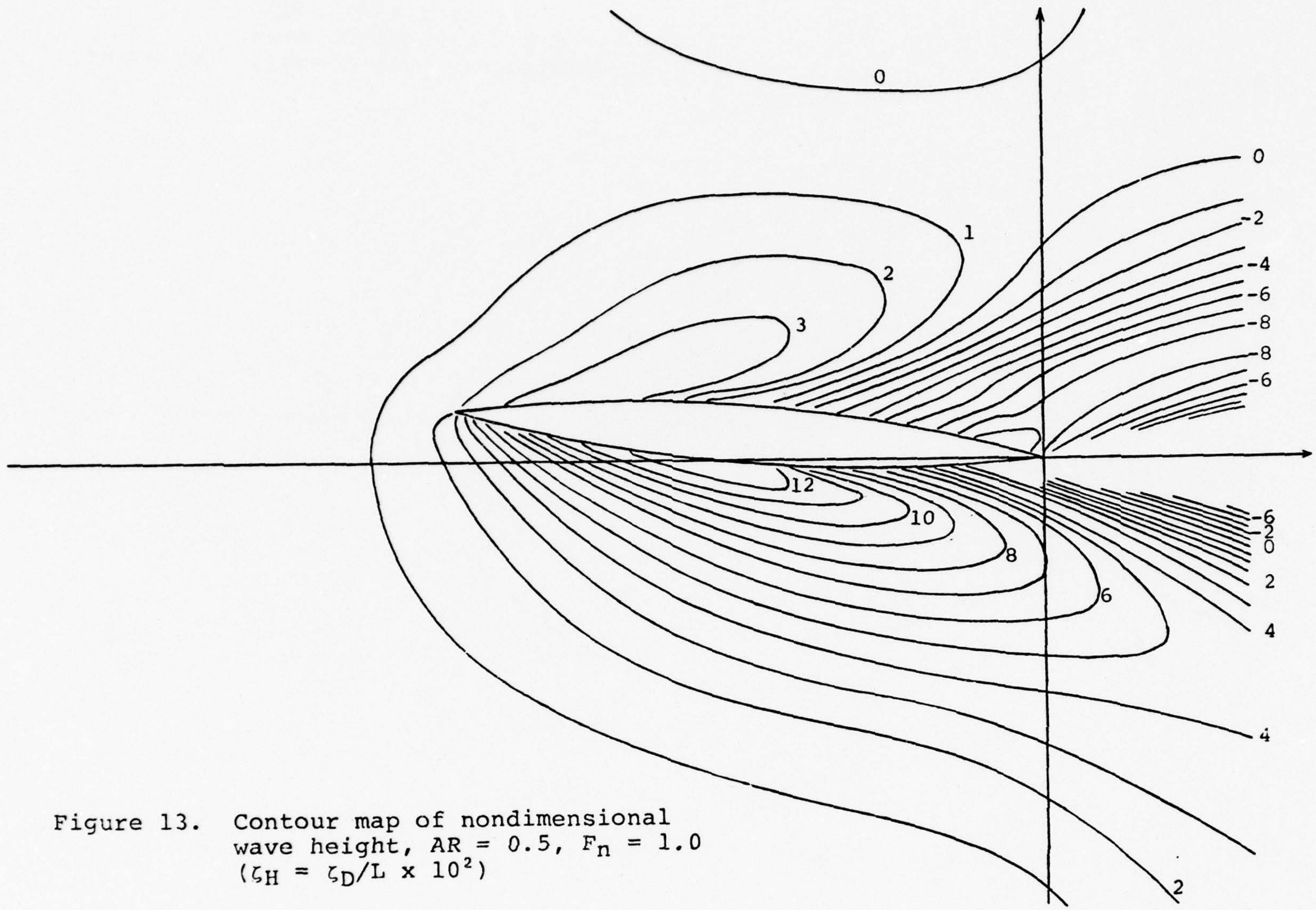

Figure 13. Contour map of nondimensional wave height, AR = 0.5, F_n = 1.0 ($\zeta_H = \zeta_D/L \times 10^2$)

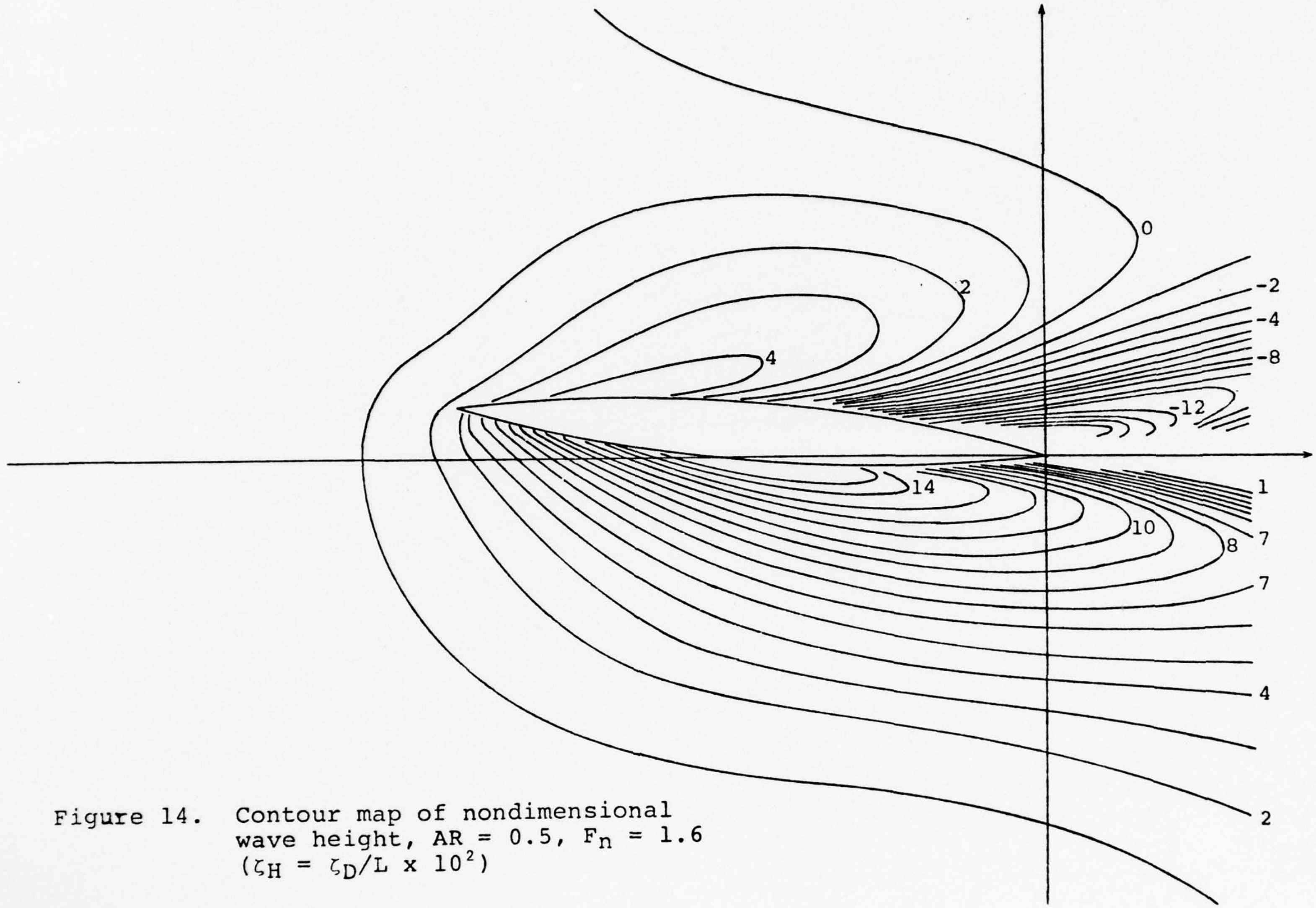

Figure 14. Contour map of nondimensional wave height, AR = 0.5, $F_n = 1.6$ ($\zeta_H = \zeta_D/L \times 10^2$)

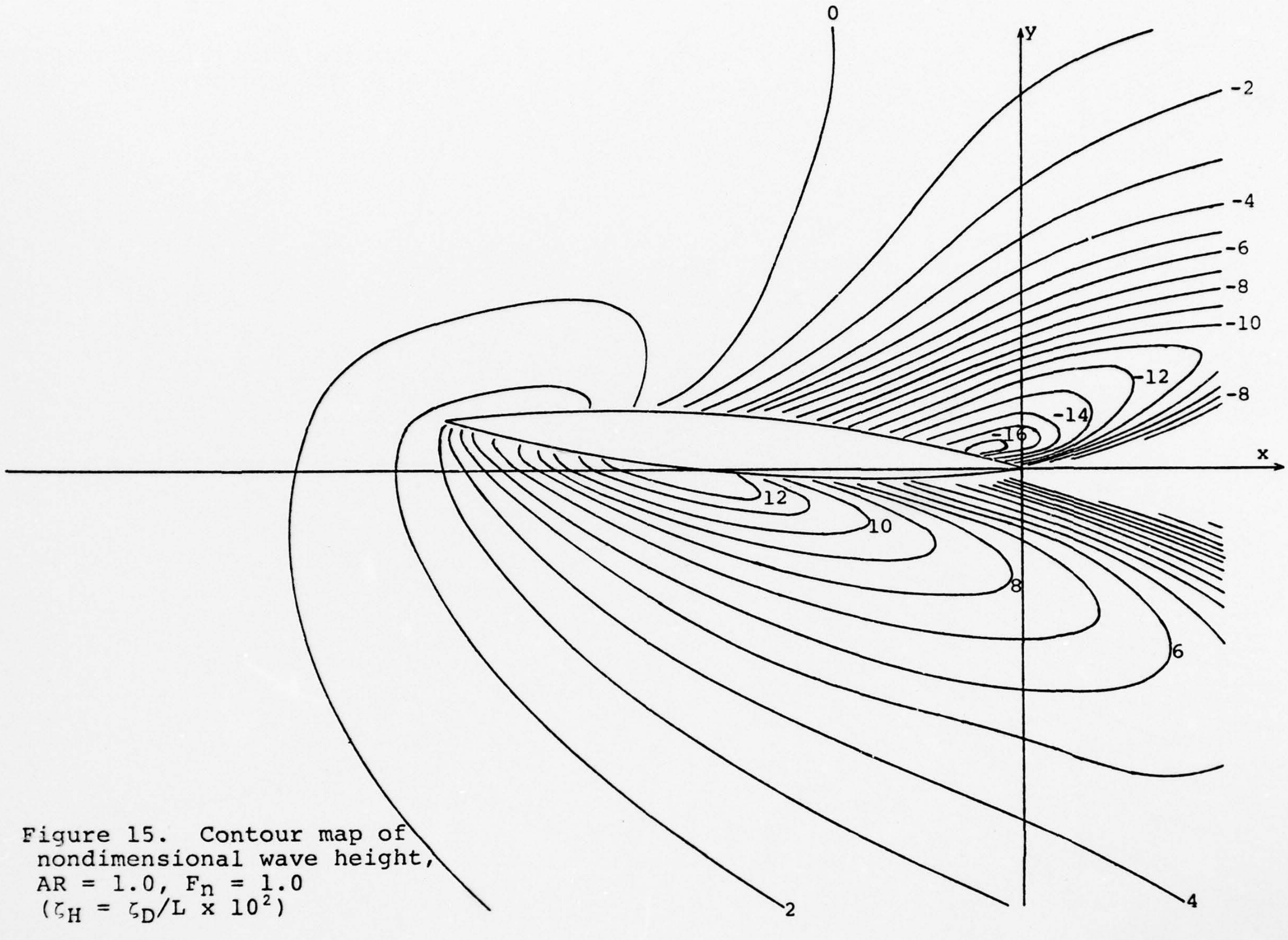

Figure 15. Contour map of nondimensional wave height, AR = 1.0, F_n = 1.0 ($\zeta_H = \zeta_D/L \times 10^2$)

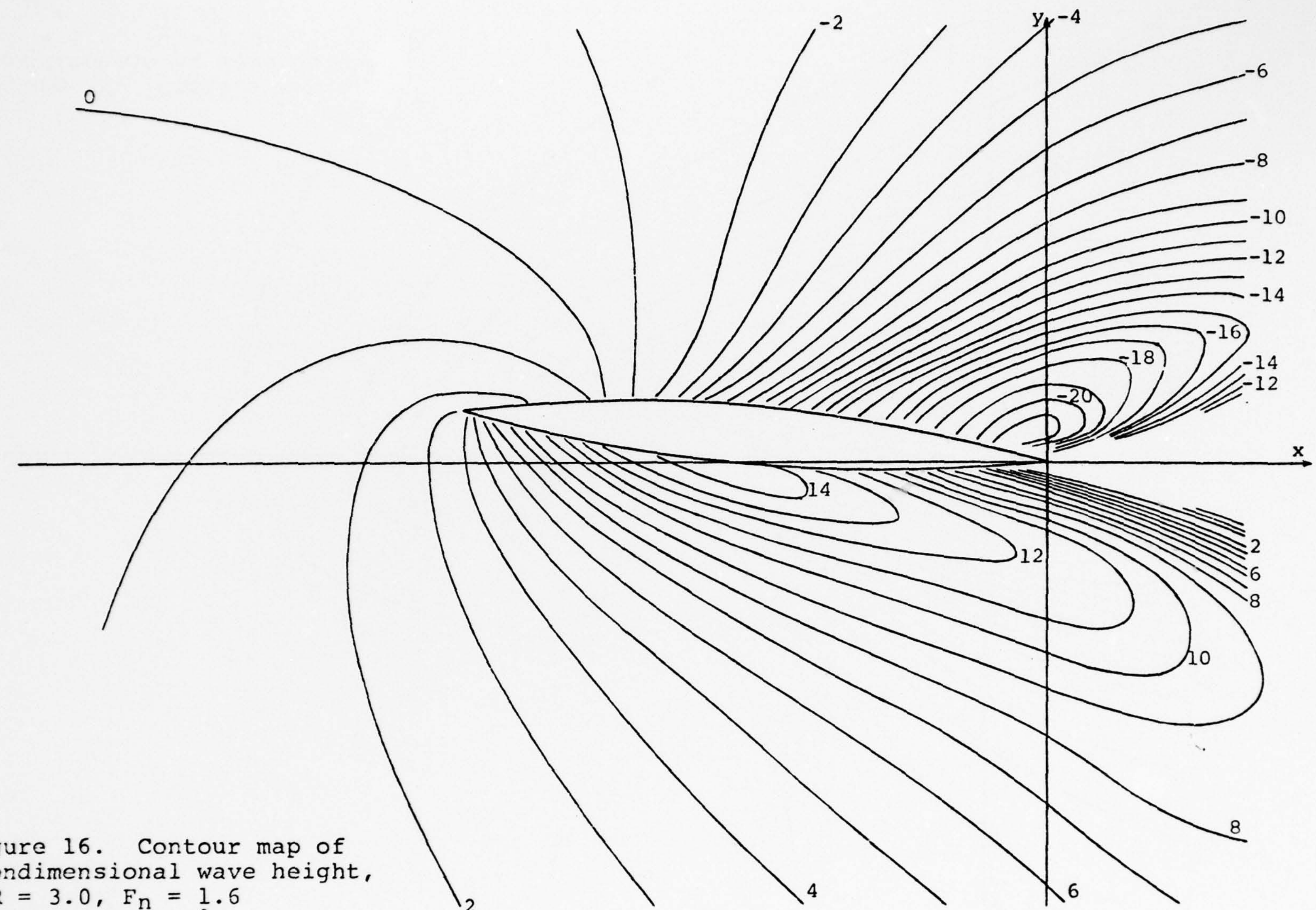

Figure 16. Contour map of nondimensional wave height, AR = 3.0, $F_n = 1.6$ ($\zeta_H = \zeta_D/L \times 10^2$)

created by the strut. The diffraction of the waves by the body can be noticed in those figures. Another feature worth noticing is that the wave pattern changes much on the suction side of the strut as the aspect ratio changes while it changes little on the pressure side of the strut. Also the disturbances ahead of the leading edge increase with the increase of the aspect ratio.

The present method can be applied for predicting the wave pattern near the bow of a ship.

Standing (1974) measured wave patterns for a model shown in Figure 17. Present theory was applied also to this

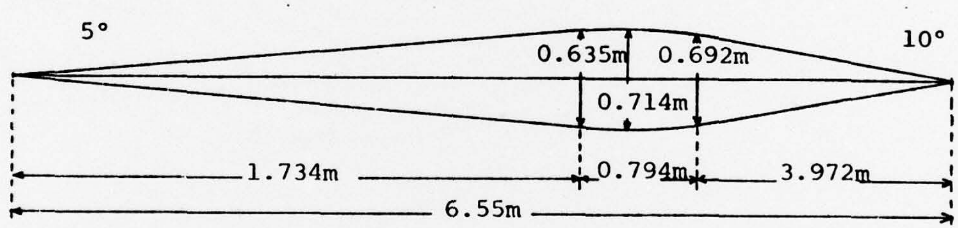

Figure 17. The double-wedge model in a plan view

case and was compared with Standing's experimental result in Figure 18. The present theory shows good agreement with experimental result in the region very near the bow. The improvement is significant compared with the time-stepping method shown in Figure 19, calculated by Daoud (1975). In the latter, disturbances upstream (as given in the present ϕ_0 problem) are not considered.

From the results shown in this chapter, it can be concluded that the present theory succeeded in including

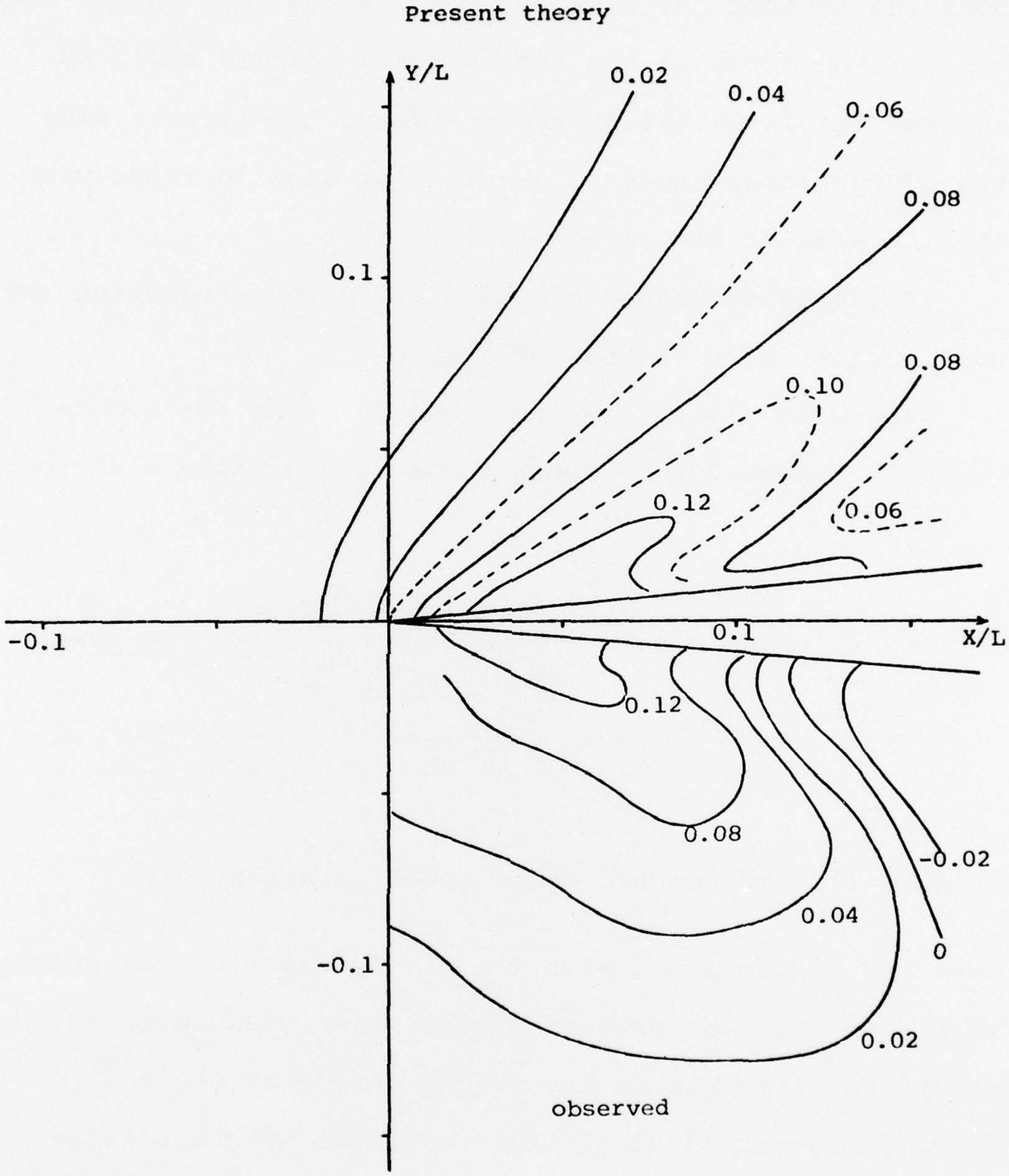

Figure 18. Contour map of wave height, $g\zeta/U^2$, present method compared with measurements by Standing (1974) (U=1.83 m/sec, F_n=0.23)

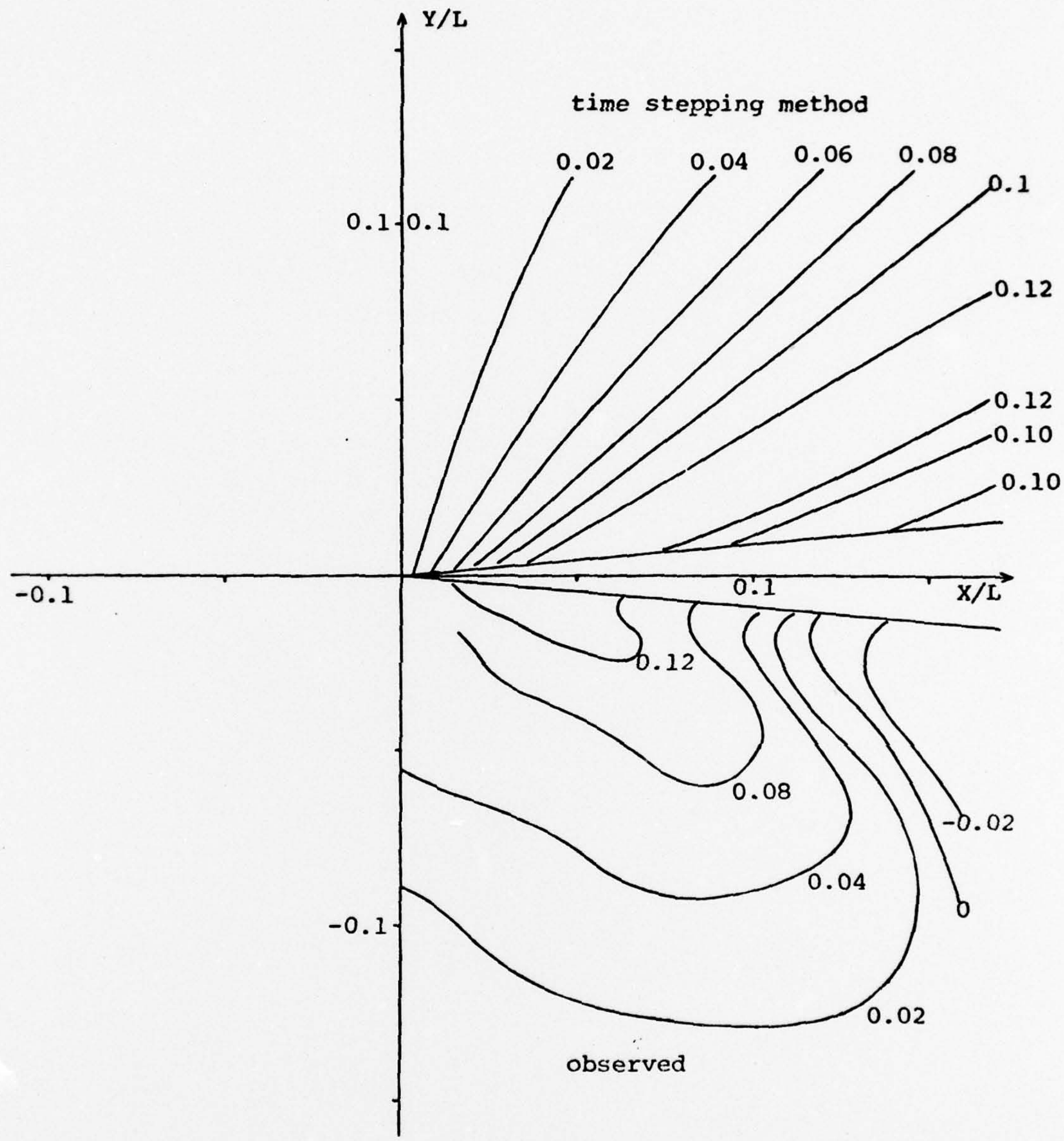

Figure 19. Contour map of wave height, $g\zeta/U^2$, Daoud's (1975) method compared with measurements by Standing (1974). (U=1.83 m/sec, F_n=0.23)

the disturbance ahead of the leading edge and the effects of the aspect ratio, and hence it was developed in the right direction.

APPENDIX DOUBLE BODY PROBLEM

The method used for solving the ϕ_0 problem can be applied to a lifting-body problem in an infinite fluid. It should be noted that the present method has an advantage that the solution of a lifting-body problem which satisfies the body-boundary condition on the actual surface of the body can be obtained by solving a Fredholm integral equation of the second kind. Also it can be applied to a body with sharp leading edge. In this appendix, the present method will be applied to a double-body problem.

Let ϕ be the nondimensional perturbation velocity potential which is the solution of the double-body problem. Then ϕ satisfies the following condition:

$$\phi_z = 0 \qquad \text{on} \qquad z = 0 \; . \tag{A1}$$

Assume the Green function as follows:

$$G(P;Q) = \frac{1}{r(P;Q)} + \frac{1}{r'(P;Q)}$$

where r and r' have the same definitions as in Equation (31). Then

$$G_z = 0 \qquad \text{on} \qquad z = 0 \; . \tag{A2}$$

Assume the same coordinate system and control surfaces as in the ϕ_0 problem. According to Green's formula, ϕ at a point $P(x,y,z)$ in V can be written:

$$\phi(P) = \frac{1}{4\pi} \iint_{ST} \{\phi(Q)\, G_n(P;Q) - \phi_n(Q)\, G(P;Q)\} dS. \tag{A3}$$

Using the conditions (A1), (A2) and following the argument as in Chapter III, the equation (A3) can be reduced to:

$$\phi(P) = \frac{1}{4\pi} \iint\limits_{H} \{\phi(Q)\ G_n - \phi_n(Q)\ G\}\ dS$$

$$+ \frac{1}{4\pi} \int_{-AR}^{0} d\rho\Gamma(\rho) \int_{0}^{\infty} G_n\ d\xi\ . \tag{A4}$$

Finally ϕ on H is the solution of an integral equation:

$$\phi = \frac{1}{2\pi} \iint\limits_{H} (\phi\ G_n - \phi_n\ G)\ dS$$

$$+ \frac{1}{2\pi} \int_{-AR}^{0} d\ \rho\Gamma(\rho) \int_{0}^{\infty} G_\eta\ d\xi\ . \tag{A5}$$

The integral equation (A5) is identical to the integral equation for ϕ_0 problem except for the plus-minus change in the Green function. Hence the mathematical method and the computer program developed for solving the ϕ_0 problem can be used for solving the double body problem by only changing the sign in the Green function. The solution of the double body problem can be checked by various experimental or theoretical results of wing theory. Hence the theory, the computer program and, indirectly, the solution of the ϕ_0 problem can be certified. This is desirable since the ϕ_0 problem is rather an artificial one and hence cannot be checked by comparing with experimental results.

The numerical calculation was performed for the strut shown in Figure 5. A numerical scheme depending on the

finite element method was used. In Figure A1 and Figure A2 are shown the distributions of nondimensional perturbation velocity potential over the midsection contour for various aspect ratios. In symmetric cases, the solutions should coincide with the two-dimensional solution on the midsection contour for any aspect ratio, since this is a double-body problem. This is clearly shown in Figure A1. In the asymmetric cases shown in Figure A2, the solutions calculated approach the analytically obtained two-dimensional solution as the aspect ratio increases. The solutions also show infinite perturbation velocity along the sharp leading edge. The calculated side force coefficient is plotted vs. aspect ratio in Figure A3. The side force coefficients for rectangular flat plates calculated by Daoud (1973) and Kerwin and Openheim (1974) based on thin wing theory are also shown in Figure A3 as well as the experimental results for rectangular wings from Thwaites (1960: p. 343).

From the results shown in these figures, it can be concluded that, adopting the present method, reasonable results can be obtained using rather a small number of elements in a numerical calculation. It seems the accuracy of the present numerical solution is not enough for discussing the effects of the thickness of the strut or the shape of the bottom; however, the accuracy of the solution can be improved to any required degree by increasing the number of the elements.

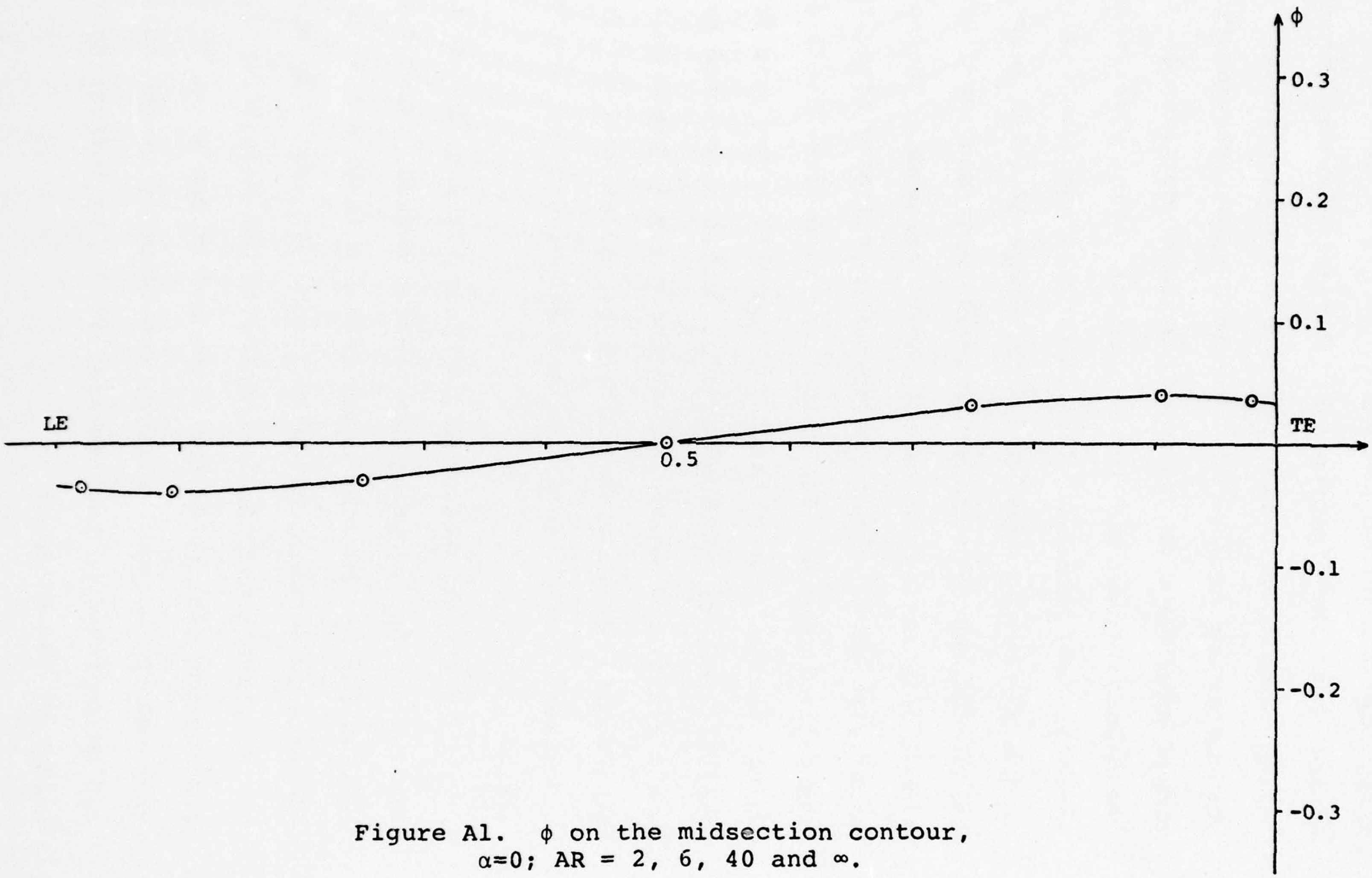

Figure A1. ϕ on the midsection contour, $\alpha=0$; AR = 2, 6, 40 and ∞.

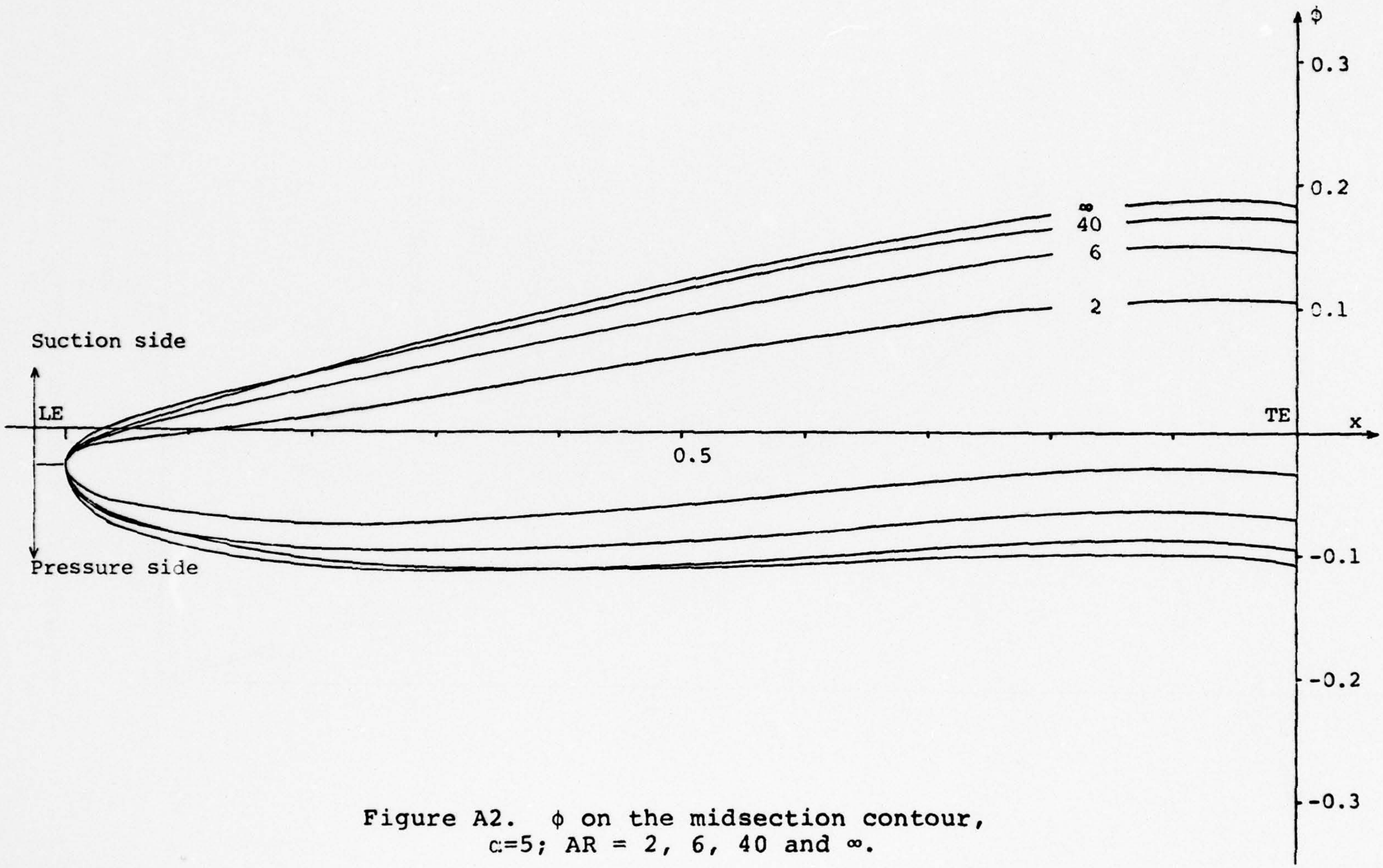

Figure A2. ϕ on the midsection contour, $\alpha=5$; AR = 2, 6, 40 and ∞.

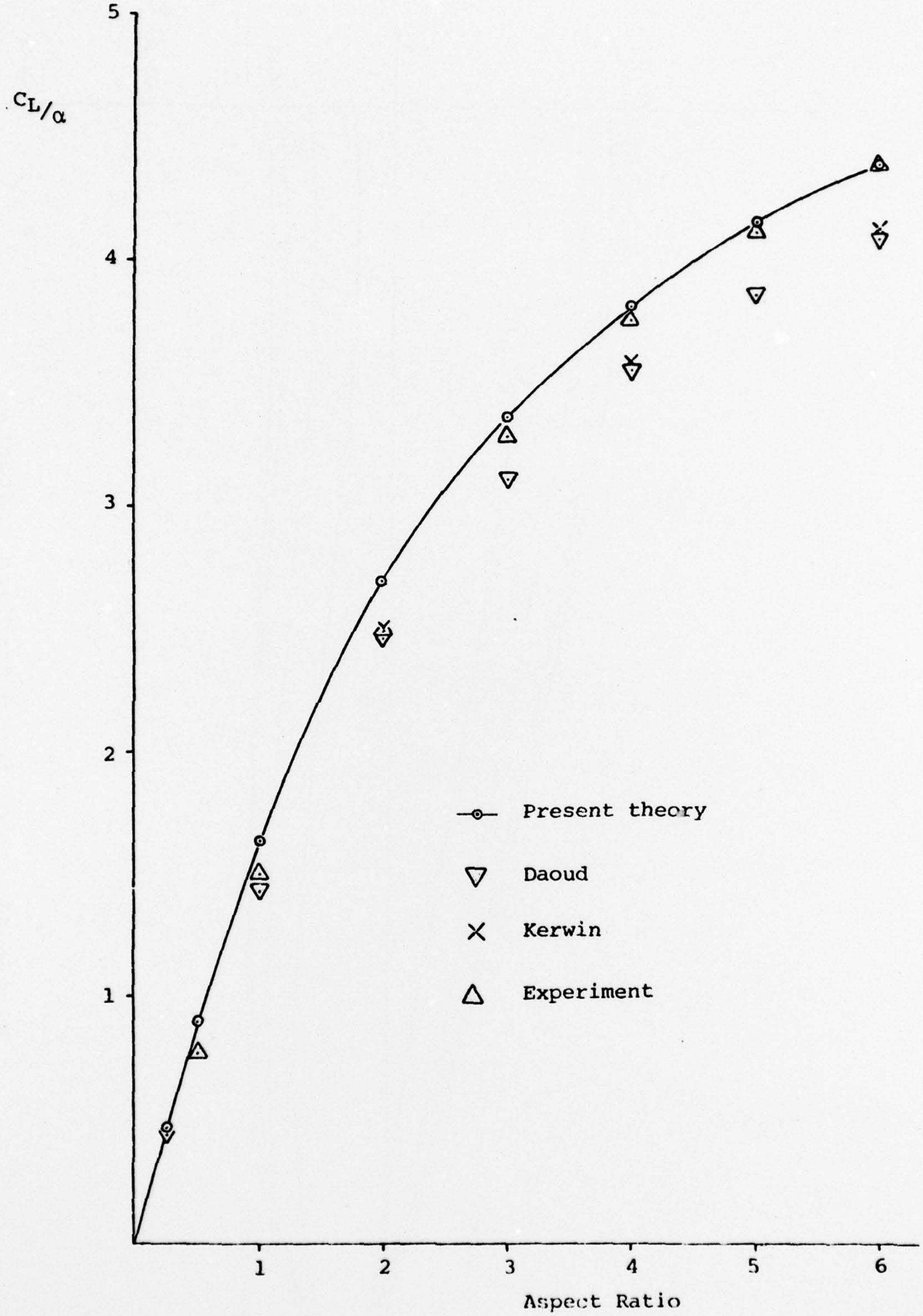

Figure A3. Side-force coefficients on rectangular wings

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